What Throttles a Flow Is Not the Thin Edge but the Thin Cut

— Raising each edge of a seven-edge network by one unit of capacity increases the maximum flow for only one of them, and raising the thinnest edge (capacity 1) changes nothing — the separator is whether one asks about a thin edge or a thin cut — [Paper 838]

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— *From thinking to syncing.* —

Abstract

It is said that the narrowest place sets the flow.This paper counts that the “place” is not an edge but a cut.No new theorem or law is claimed. First, the maximum flow and the smallest cut found by trying every cut agree——in all four networks two unrelated computations returned the same number (Section 2). **Second, and this is the core. Raising the thinnest edge can change nothing——in a seven-edge network, raising the capacity-1 edge by one leaves the flow at 7** (Section 3). **Third, very few edges help——only 1 of the 7 helped, and there were 3 minimum cuts** (Section 3). **Fourth, with thin edges in series, none helps alone——all 4 edges give an increase of 0, and only raising both thin edges takes 4 to 5** (Section 3). **Fifth, the same thinness in parallel does help——where two thin edges sit in one cut, raising either alone takes 8 to 9** (Section 3). **Sixth, the separator is whether one asks about a thin edge or a thin cut——an edge’s thinness can simply be looked at; a cut is invisible until one decides where to cut** (Section 4).
Keywords: maximum flow, minimum cut, bottleneck, cut, series and parallel, model.

Scope of this paper (scope note): **No new theorem or law is claimed——that the maximum flow equals the capacity of a minimum cut, that augmenting paths find the maximum flow, and that raising a saturated edge leaves the flow unchanged when another cut remains, are all known.No real network is claimed——the capacities and shapes are a model built for counting; they match no pipe and no circuit.The theorem is not proved——the max-flow min-cut theorem is used as known, and cuts are enumerated in small networks.Speed of computation is not discussed——which algorithm is faster is not the subject.No design advice is given——where to spend money is not stated.Relation to earlier papers**: **Paper 540 showed that the upper and lower bounds of a composite are the arithmetic and harmonic means themselves——this paper stands beside it: there, placing things in series let the weaker one dominate; here, placing them in series means neither weak one helps until both are fixed.Paper 300 showed that whether two things share a root is decidable——a thin edge and a thin cut are distinct roots by that criterion.**What is added is **laying out the increase for every edge raised one at a time**, **building a network where the thinnest edge gives an increase of 0**, **comparing series against parallel at the same thinness and finding the answers reversed**, and **placing the separator on thin edge versus thin cut**.

1. Notation and the Question

Setting: **a collection of directed edges with capacities is a network, and the largest amount sendable from a source s to a sink t is the maximum flow.Choosing a set S containing s but not t, the capacity of the cut is the sum of the capacities of the edges leaving S.**

$$\text{maximum flow} = \min_{S \ni s,\ S \not\ni t} \sum_{u \in S,\ v \notin S} c(u, v)$$

Here the maximum flow is found by augmenting paths, and the smallest cut by trying every choice of S.The two computations share nothing.

Question: **which edge, if widened, increases the flow?**

2. Two Computations Agree

Computation 2.1: **for four networks, the maximum flow and the smallest cut over all choices were computed separately.**

network	maximum flow	smallest cut	number of smallest cuts
single path	3	3	1
thin edges in parallel	8	8	1
thin edges in series	4	4	2
thin edge off to the side	7	7	3

> **Observation 2.2**
>
> **all four agree.One searches for paths and pushes; the other partitions the vertices every way and adds, and nothing in between is shared——so the agreement is not a copy of either.**

> **Observation 2.3**
>
> **the smallest cut need not be unique.The series network has 2, the one with a thin edge off to the side has 3 ——that count decides what happens in the next section.**

3. Edges That Help and Edges That Do Not

Computation 3.1: **each edge was raised by one unit of capacity in turn, and the increase in the maximum flow recorded.**

network	flow	edges	edges that helped	increase at the thinnest edge
single path	3	3	1	1
thin in parallel	8	6	2	1
thin in series	4	4	0	0
thin off to the side	7	7	1	0

Observation 3.2

this is the core. There are networks where the thinnest edge does not help.With a thin edge off to the side, raising the capacity-1 edge leaves the flow at 7, and only one other edge helps——that edge lies in no cut, so cutting it or not makes no difference to any capacity.

Computation 3.3: **in the series network every one of the four increases was 0. Raising both thin edges changes that.**

what was done	maximum flow
nothing	4
one thin edge raised	4
both thin edges raised	5

Observation 3.4

with two minimum cuts, no single edge helps.Raising one leaves the other cut at the same capacity——"no single edge helps" and "none of them is needed" are different statements.

Computation 3.5: **at the same thinness (two edges of capacity 4), placing them in parallel reverses the outcome.**

placement of the thin edges	number of smallest cuts	edges that help alone
series	2	0
parallel	1	2

Observation 3.6

the same edge thinness yields opposite answers.In parallel the two share one cut, so raising either raises that cut's capacity——looking only at edges, the two cases cannot be told apart.

4. The Separator

> **Observation 4.1**
>
> **the separator is whether one asks about a thin edge or a thin cut.An edge's thinness can be read off one edge at a time; a cut is invisible until one decides where to cut——the first is a local quantity, the second a quantity about partitions.**

> **Observation 4.2**
>
> **placed beside Paper 540.540 showed that the bounds of a composite are the arithmetic and harmonic means themselves, and counted how series placement lets the weaker one dominate——there series made the weaker one the answer; here series means neither weak one helps until both are fixed. The same word "series" yields different consequences.Paper 300 showed that whether two things share a root is decidable ——a thin edge and a thin cut are distinct roots by that criterion.**

> **Observation 4.3**
>
> **what is not said.Real networks, algorithmic speed and cost are not treated——what is shown is that the quantity that grows belongs to a cut, not to an edge.**

5. Census and Closing

1. **two computations agree in all four** (Computation 2.1): **nothing in between is shared.**
2. **smallest cuts number 1, 1, 2, 3** (Computation 2.1): **not always unique.**
3. **the capacity-1 edge gives 0** (Computation 3.1): **the core.**
4. **1 of 7 edges helps** (Computation 3.1): **few edges help.**
5. **series: all 4 give 0** (Computation 3.3): **none helps alone.**
6. **both together take 4 to 5** (Computation 3.3): **not that none is needed.**
7. **parallel: either takes 8 to 9** (Computation 3.5): **same thinness, reversed.**
8. **no edge ever lowers the flow** (Computation 3.1): **increases are non-negative.**
9. **a cut is about partitions** (Observation 4.1): **an edge is local.**

10. **thin edge, or thin cut** (Observation 4.1): **the separator; Papers 540, 300.**

Closing: **What throttles a flow is not the thin edge but the thin cut.Raising each edge of a seven-edge network by one unit increases the flow for only one of them, and the thinnest edge (capacity 1) gives an increase of 0——because that edge lies in no cut at all.With two thin edges in series there are two minimum cuts, all four increases are 0, and only raising both takes 4 to 5——because raising one leaves the other cut at the same capacity.At the same thinness in parallel there is a single cut holding both, and either edge alone takes 8 to 9.Placed among the earlier papers——beside Paper 540's "in series the weaker one dominates", and treating thin edge and thin cut as distinct roots by the criterion of Paper 300.To be honest——the networks here are a model built for counting, matching no pipe and no circuit.**

References

1. Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to Algorithms*. MIT Press.
2. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
3. Yamagishi, Y. (2026). *The Upper and Lower Bounds of a Composite Are the Arithmetic and Harmonic Means Themselves* [Paper 540]. Zenodo. 10.5281/zenodo.23014138

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