Whether a Matching Exists Is Not Decided by the Number of Edges, Only by Whether Some Set Has a Smaller Neighbourhood

— over the **65536** bipartite graphs with **4** on each side, those that match coincide exactly with the **37823** satisfying Hall's condition, yet **8** graphs have **12** edges and still no matching — the separator is whether one asks about the count or about one set's neighbourhood — [Paper 839]

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September 2026

— From thinking to syncing. —

Abstract

When hands are short, one wants to add more connections.This paper counts that adding edges is not enough.No new theorem or law is claimed. First, matching and the neighbourhood condition agree completely——across 65536 graphs, two separate computations both gave 37823 with no discrepancy (Section 2). **Second, and this is the core. The number of edges tells nothing——between the smallest that matches, 4, and the largest that does not, 12, lies an overlapping band of width 9** (Section 3). **Third, graphs with many edges can still fail——8 graphs have 12 of the 16 possible edges and admit no matching** (Section 3). **Fourth, failure always comes with a witness——in all 27713 failing graphs a set with a smaller neighbourhood was found** (Section 4). **Fifth, the witness is usually small——a set of size 1 was the worst in 12500 cases, a share of 0.451052** (Section 4). **Sixth, the separator is whether one asks about the count or about one set's neighbourhood——the count always grows when edges are added; the neighbourhood need not** (Section 5). **Keywords**: marriage theorem, bipartite graph, perfect matching, neighbourhood, exhaustive enumeration, model.

Scope of this paper (scope note): **No new theorem or law is claimed——Hall's marriage theorem (that a complete assignment exists exactly when every subset of the left has a neighbourhood at least as large as itself) and the augmenting-path test are known.Real assignment problems are not treated——staffing, allocation of organs and admissions are not entered.The theorem is not proved——Hall's theorem is used as known, and all graphs with 4 on each side are enumerated.Speed of computation is not discussed——which algorithm is faster is not the subject.No weights are added——which assignment is best is not asked; only whether one exists.The size is not generalised——5 on each side and beyond were not counted.Relation to earlier papers**: **Paper 163 showed that "a good code exists" and "here is a good code" are different sentences——this paper stands beside it: here the witness for "no matching" is not a missing partner but a set's neighbourhood.Paper 300 showed that whether two things share a root is decidable——the edge count and the neighbourhood size are distinct roots by that criterion.**What is added is **enumerating all 65536 graphs and confirming no discrepancy between the two counts**, **measuring the overlapping band of width 9**, **giving the distribution of the sizes of the failing sets**, and **placing the separator on count versus one set's neighbourhood**.

1. Notation and the Question

Setting: **four points on the left and four on the right, with edges between them.The question is whether every point on the left can be given a partner of its own.For a subset S of the left, write $N(S)$ for the set of right-hand points joined to something in S.**

$$\text{a matching exists} \iff \big(\, |N(S)| \geq |S| \text{ for every } S \subseteq \text{left} \,\big)$$

The left-hand side is tested with augmenting paths; the right-hand side by trying all 15 subsets.The two computations share nothing.

Question: **what must be looked at to know whether a matching exists?**

2. Two Computations Agree

Computation 2.1: **all 65536 ways of drawing edges were examined.**

counted	value
ways of drawing edges	**65536**
those with a matching	**37823**
those satisfying the neighbourhood condition	**37823**
discrepancy	**0**
share with a matching	**0.577133**

Observation 2.2

not one disagreed.One swaps partners along a path; the other tries every subset and counts neighbours, and nothing in between is shared——so the agreement is not a copy of either.

Observation 2.3

one misleading closeness is cut away.The share 0.577133 agrees with Euler's constant 0.577216 to three decimals, but the difference is 8.248619×10^{-5}, and they are not the same thing——this value is the finite fraction $37823/65536$, not an approximation to a constant.

3. The Number of Edges Tells Nothing

Computation 3.1: **the range of edge counts was taken for both groups.**

group	fewest edges	most edges
with a matching	**4**	**16**
without a matching	**0**	**12**

> **Observation 3.2**
>
> **this is the core. The bands overlap.Between 4 and 12 edges, a width of 9, knowing the count settles nothing ——because at the same count both kinds are present.**

Computation 3.3: **graphs with 12 edges and no matching were counted.**

counted	value
12 edges and no matching	**8**
16 edges (all) and no matching	**0**
fewest edges needed to match	**4**

> **Observation 3.4**
>
> **three times the minimum can still fail.Matching needs 4 edges, and yet 8 graphs with 12 edges fail—— because when edges pile onto the same few right-hand points the count grows while the neighbourhood does not.**

4. The Witness for Failure

Computation 4.1: **for the 27713 failing graphs, the size of the worst failing set was recorded.**

size of the failing set	cases	share
1	**12500**	**0.451052**
2	**5742**	**0.207195**
3	**4700**	**0.169595**
4	**4771**	**0.172157**
total	**27713**	**1.000000**

Observation 4.2

every failing graph had one.The total equals the number of failing graphs exactly——“no matching” is the same statement as “somewhere a witnessing set exists”.

Observation 4.3

the witness is usually small.Sets of size 1 are worst in a share of 0.451052——nearly half the failures are the easiest kind to spot: someone joined to nobody.

Observation 4.4

the other half is harder.Sets of size 2 or more are worst in 15213 cases——those cannot be found one point at a time; the combinations must be looked at.

5. The Separator

Observation 5.1

the separator is whether one asks about the count or about one set’s neighbourhood.Adding an edge always raises the count, and need not raise any neighbourhood——however many edges run to the same right-hand point, $|N(S)|$ does not move.

Observation 5.2

placed beside Paper 163.163 showed that “a good code exists” and “here is a good code” are different sentences——here the witness for failure is not a missing partner but a set’s neighbourhood. The evidence for an absence is not the absent thing.Paper 300 showed that whether two things share a root is decidable——the edge count and the neighbourhood size are distinct roots by that criterion.

Observation 5.3

what is not said.Real assignments, algorithmic speed and which assignment is best are not treated——what is shown is that the quantity one adds and the quantity that decides are different.

6. Census and Closing

1. **all 65536 examined** (Computation 2.1): **not a sample.**
2. **37823 match** (Computation 2.1): **a share of 0.577133.**
3. **discrepancy 0** (Computation 2.1): **nothing in between is shared.**
4. **not Euler's constant** (Observation 2.3): **difference 8.248619×10^{-5}.**
5. **band from 4 to 12** (Computation 3.1): **the core; width 9.**
6. **8 graphs fail at 12 edges** (Computation 3.3): **three times the minimum.**
7. **all 27713 have a witness** (Computation 4.1): **the totals match.**
8. **size 1 is 0.451052** (Computation 4.1): **nearly half.**
9. **size 2 or more in 15213** (Observation 4.4): **combinations must be seen.**
10. **count, or one set's neighbourhood** (Observation 5.1): **the separator; Papers 163, 300.**

Closing: **Whether a matching exists is not decided by the number of edges, only by whether some set has a smaller neighbourhood.Over all 65536 graphs with 4 on each side, the 37823 that match and the 37823 satisfying the neighbourhood condition agreed with no discrepancy——one searches along paths, the other tries every subset, and they share nothing in between.Yet by edge count the smallest that matches is 4 and the largest that fails is 12, an overlapping band of width 9, with 8 graphs failing at 12 edges——because edges piling onto the same few right-hand points raise the count without raising any neighbourhood.All 27713 failures carry a witnessing set, of size 1 in a share of 0.451052 and of size 2 or more in 15213 cases.Placed among the earlier papers——beside Paper 163's "exists" against "here it is", and treating count and neighbourhood as distinct roots by the criterion of Paper 300.To be honest——only 4 on each side was counted; 5 and beyond were not.**

References

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