A Tree Has More Than One “Middle”: In Over Half of All Trees the Centre and the Centroid Are Different Points

— Building all 16807 trees on 7 vertices and all 262144 on 8, the farthest vertex from any starting point is always an end of a diameter (a share of 1.000000) — the separator is whether one wants to shorten the farthest reach or to shrink the largest piece — [Paper 840]

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— *From thinking to syncing.* —

Abstract

The “middle” of a tree sounds like one thing.This paper counts that it is two, and that they part in over half of all trees.No new theorem or law is claimed. First, in a tree, walking twice finds the two farthest vertices——across all 16807 trees on 7 vertices and all 262144 on 8, no starting point ever failed (Section 2). **Second, this holds because they are trees——among the 26704 connected graphs on 6 vertices, 4897 (0.183381) break it** (Section 2). **Third, not one of the failures is a tree——of those 4897, exactly 0 are trees** (Section 2). **Fourth, and this is the core. There are two “middles”——centre and centroid differ in 9030 trees on 7 vertices (0.537276) and 140616 on 8 (0.536407)** (Section 3). **Fifth, and yet they never stand far apart——at most a distance of 1 on 7 vertices and 2 on 8** (Section 3). **Sixth, the separator is whether one wants to shorten the farthest reach or to shrink the largest piece——one word “middle” answers two different questions** (Section 4). **Keywords**: tree diameter, eccentricity, centre and centroid, breadth-first search, exhaustive enumeration, model.

Scope of this paper (scope note): **No new theorem or law is claimed——that two breadth-first searches give the diameter of a tree, that a tree's centre consists of one or two vertices (Jordan), and that the same holds for the centroid, are all known.No real tree is claimed——no wiring, organisation chart or phylogeny is matched; all labelled trees of a given size were simply built and counted.The theorem is not proved——that two searches work on a tree is used as known and confirmed by exhaustive enumeration.Speed of computation is not discussed——which algorithm is faster is not the subject.No weights are added——trees with edge lengths are not treated; every edge has length 1 .The size is not generalised——9 vertices and beyond were not counted.Relation to earlier papers**: **Paper 182 showed that "the curse of dimensionality" is not one curse——this paper does not sit in that seat but stands beside it: there one name pointed at several difficulties, here one word points at two vertices.Paper 300 showed that whether two things share a root is decidable——centre and centroid are distinct roots by that criterion.**What is added is **building every tree on 7 and 8 vertices and confirming that two searches never fail**, **measuring the failure share 0.183381 on connected graphs and confirming no tree among them**, **measuring how often centre and centroid part and how far**, and **placing the separator on farthest reach versus largest piece**.

1. Notation and the Question

Setting: **consider trees in which every edge has length 1.The eccentricity of a vertex v is the distance to the vertex farthest from it.The diameter is the largest eccentricity.**

$$\operatorname{ecc}(v) = \max_{u} d(v,u), \qquad \text{diameter} = \max_{v} \operatorname{ecc}(v), \qquad \text{centre} = \arg\min_{v} \operatorname{ecc}(v)$$

Separately, write $w(v)$ for the size of the largest piece left when v is removed; the vertices minimising w are the centroid.

Question: **which of these is the "middle" of a tree?**

2. Walking Twice Finds an End

Computation 2.1: **all labelled trees were built, and from every vertex it was checked whether the farthest vertex is an end of a diameter.**

vertices	trees	share where two searches gave the diameter
6	1296	1.000000
7	16807	1.000000
8	262144	1.000000

> **Observation 2.2**
>
> **not one failed.It holds for every tree and every starting vertex——this is not a sample but all trees of that size.**

Computation 2.3: **to doubt the "1.000000", the same procedure was run on graphs that are not trees: all connected graphs on 6 vertices.**

counted	value
connected graphs on **6** vertices	**26704**
those where two searches miss the diameter	**4897**
that share	**0.183381**
of those, how many are trees	**0**

> **Observation 2.4**
>
> **so the "1.000000" is not an empty swing.The same procedure fails on a share of 0.183381 once cycles are allowed——the failures show from the other side that being a tree is exactly what is doing the work.**

3. Two "Middles"

Computation 3.1: **the centre (least eccentricity) and the centroid (smallest largest piece) were compared across all trees.**

vertices	trees where they differ	share	largest distance between them
6	**480**	**0.370370**	**1**
7	**9030**	**0.537276**	**1**
8	**140616**	**0.536407**	**2**

Observation 3.2

this is the core. They differ in over half.The share is 0.537276 on 7 vertices and 0.536407 on 8——so in over half of all trees it matters which “middle” was meant.

Computation 3.3: the number of centre vertices was counted.

vertices	centre of size 1	centre of size 2
6	726	570
7	8617	8190
8	127688	134456

Observation 3.4

the centre never has three vertices.At every size only 1 or 2 appeared——because the middle of a longest path is either a vertex or an edge.

Observation 3.5

and yet they never stand far apart.The largest distance is 1 on 7 vertices and 2 on 8——“different” and “far” are not the same. They differ and they are neighbours.

4. The Separator

Observation 4.1

the separator is whether one wants to shorten the farthest reach or to shrink the largest piece.The first helps the worst-placed vertex; the second evens out how the tree splits——both are called the “middle”, and they answer different questions.

Observation 4.2

placed beside Paper 182.182 showed that “the curse of dimensionality” is not one curse——there one name pointed at several difficulties; here one word points at two vertices of the same tree. The difference is whether the multiplicity sits in the name or in the place.Paper 300 showed that whether two things share a root is decidable——centre and centroid are distinct roots by that criterion: moving one leaves the other where it was.

Observation 4.3

what is not said.Real trees, algorithmic speed and trees with edge lengths are not treated——what is shown is that the “middle” becomes a different vertex depending on the question.

5. Census and Closing

1. **two searches give the diameter** (Computation 2.1): **a share of 1.000000.**
2. **confirmed on 262144 trees** (Computation 2.1): **not a sample.**
3. **connected graphs fail at 0.183381** (Computation 2.3): **the 1.000000 was doubted.**
4. **0 of the failures are trees** (Computation 2.3): **the condition is doing the work.**
5. **0.537276 on 7 vertices** (Computation 3.1): **the core.**
6. **0.536407 on 8** (Computation 3.1): **still over half.**
7. **the centre has 1 or 2** (Computation 3.3): **never three.**
8. **distance 1 on 7 vertices** (Computation 3.1): **different but neighbours.**
9. **distance 2 on 8** (Computation 3.1): **never far.**
10. **farthest reach, or largest piece** (Observation 4.1): **the separator; Papers 182, 300.**

Closing: **A tree has more than one “middle”: in over half of all trees the centre and the centroid are different points.Across all 16807 trees on 7 vertices and all 262144 on 8, the farthest vertex from any starting point is an end of a diameter, a share of 1.000000——and that this is no empty swing is shown by the 4897 of 26704 connected graphs on 6 vertices (0.183381) where the same procedure fails, none of them a tree.Meanwhile the vertex of least eccentricity and the vertex whose removal leaves the smallest piece differ in a share of 0.537276 on 7 vertices and 0.536407 on 8——because the first helps the worst-placed vertex while the second evens out how the tree splits.Even so they are never far apart: at most 1 on 7 vertices and 2 on 8, different but neighbours.Placed among the earlier**

papers——beside Paper 182's "one name, several difficulties", in the form of one word pointing at two vertices, and treating centre and centroid as distinct roots by the criterion of Paper 300.To be honest——only up to 8 vertices was counted; 9 and beyond were not.

References

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