

Split Into Remainders, Addition and Multiplication Survive but Order Vanishes

— With moduli **3**, **5** and **7** (product **105**), every one of the **105** triples of remainders occurs exactly once, and both addition and multiplication can be done componentwise (no discrepancy in any case) — the separator is whether one asks about addition and multiplication or about order — [Paper 842]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/842>

— *From thinking to syncing.* —

Abstract

Replacing a large number by a tuple of small remainders makes arithmetic easier. This paper counts what survives that replacement and what does not. No new theorem or law is claimed. First, with coprime moduli every triple occurs exactly once: **105** numbers give **105** triples, with no overlap and nothing missing (Section 2). Second, addition and multiplication can both be done componentwise: across 105×15 pairs there was no discrepancy at all (Section 2). Third, and this is the core. Order is almost unreadable: only **1260** of **10920** pairs (**0.115385**) have all three remainders pointing the same way (Section 3). Fourth, and even then it misleads: **372** of those **1260** (**0.295238**) point against the true order (Section 3). Fifth, without coprimality fewer triples occur: with **(6, 10, 15)** only **30** of a product of **900** occur (**0.033333**) (Section 4). Sixth, the separator is whether one asks about addition and multiplication or about order: one representation is sufficient for one question and says nothing about another (Section 5). **Keywords:** Chinese remainder theorem, remainder, coprime, least common multiple, comparison of size, exhaustive enumeration.

Scope of this paper (scope note): No new theorem or law is claimed: the Chinese remainder theorem, the one-to-one correspondence between numbers and tuples of remainders under coprime moduli, componentwise addition and multiplication, and the need to reconstruct before comparing in a residue number system, are all known. Cryptography is not treated: public-key schemes and implementations of residue arithmetic are not entered. Speed of computation is not discussed, which representation is faster is not the subject. The theorem is not proved: the Chinese remainder theorem is used as known, with exhaustive enumeration over small moduli. Counting the sexagenary cycle is not treated: counting how many pairs occur from the greatest common divisor of two periods belongs to Paper 605. No extension is made: negative numbers, rationals and the polynomial case are not entered. Relation to earlier papers: Paper 605 counted, from the greatest common divisor of the periods, that the sexagenary cycle has sixty pairs and not a hundred and twenty. This paper does not sit in that seat but stands beside it: there how many combinations occur, here which operations survive the split into remainders and which do not. Paper 300 showed that whether two things share a root is decidable: addition and multiplication on one side, order on the other, are distinct roots by that criterion. What is added is confirming by exhaustive enumeration that addition and multiplication never disagree, measuring that only 0.115385 of pairs have their remainders aligned, measuring that 0.295238 of those are reversed, and placing the separator on arithmetic versus order.

1. Notation and the Question

Setting: choose pairwise coprime moduli $m_1, \dots, m_k$ and replace a number x by its tuple of remainders.

$$x \mapsto (x \bmod m_1, \dots, x \bmod m_k), \quad M = \prod_i m_i$$

Here $m = (3, 5, 7)$ and $M = 105$. What can and cannot be done in that representation is settled by exhaustive enumeration.

Question: split into remainders, what survives and what vanishes?

2. What Survives

Computation 2.1: the triples produced by the 105 numbers 0 to 104 were counted.

counted	value
moduli	3, 5, 7
product	105
triples that occur	105
triples available	105

Observation 2.2

nothing overlaps and nothing is missing. The 105 numbers produce the 105 triples exactly once each, which is why the number can be recovered from its triple.

Computation 2.3: whether addition and multiplication work componentwise was checked exhaustively.

tried	discrepancies
adding componentwise and reassembling	0
multiplying componentwise and reassembling	0

Observation 2.4

both operations carry across intact. They break into three small computations that never have to consult one another about carries. This is the content of “splitting into remainders makes things easier”.

3. What Vanishes

Computation 3.1: over the 10920 ordered pairs from the 105 numbers, it was counted how often all three remainders point the same way.

counted	pairs	share
all pairs	10920	1.000000
all three remainders aligned	1260	0.115385
of those, reversed against the truth	372	0.295238

Observation 3.2

this is the core. Most pairs say nothing at all. Only 0.115385 are aligned; in the rest the three remainders point in different directions: order does not ride on any one remainder but on the position the three together fix.

Observation 3.3

and alignment is not to be trusted. Of the 1260 aligned pairs, 372 (0.295238) run against the true order: reading “all three smaller, so smaller” fails nearly one time in three.

Observation 3.4

addition and multiplication failed not once. One side gives 0 discrepancies, the other gives 0.295238 reversed: on the same representation, two questions are answered in wholly different ways.

4. When the Moduli Are Not Coprime

Computation 4.1: for other sets of moduli, the number of tuples that actually occur was counted.

moduli	product	least common multiple	tuples occurring	share
(4, 6)	24	12	12	0.500000
(4, 9)	36	36	36	1.000000
(6, 10, 15)	900	30	30	0.033333

Observation 4.2

being coprime is the same statement as every tuple occurring. The share is 1.000000 for (4, 9), 0.500000 for (4, 6) and 0.033333 for (6, 10, 15). What fixes the count is not the product but the least common multiple.

Observation 4.3

this is the side Paper 605 counted. There, for one pair of periods, the number of combinations was read off the greatest common divisor. Here the same mechanism is laid out as far as three moduli and read as a share. Not sitting in that seat but standing beside it.

5. The Separator

Observation 5.1

the separator is whether one asks about addition and multiplication or about order. The first is closed inside the remainders; the second needs the position in the whole, so to know order, one must return to the number itself.

Observation 5.2

one representation is sufficient or empty depending on the question. Whether a representation is good cannot be settled by the representation alone. It is settled only once it is known what will be asked.

Observation 5.3

by the discipline of Paper 300 these are distinct roots. Sums and products close inside the remainders; order does not: one being possible is no reason for the other.

Observation 5.4

what is not said. Cryptography, computational speed and the counting of the sexagenary cycle are not treated. What is shown is that changing a representation leaves different operations in different states.

6. Census and Closing

1. **105 numbers give 105 triples** (Computation 2.1): **no overlap, nothing missing.**
2. **addition: 0 discrepancies** (Computation 2.3): **carries across.**
3. **multiplication: 0 discrepancies** (Computation 2.3): **likewise.**
4. **aligned pairs 0.115385** (Computation 3.1): **the core.**
5. **of those, 0.295238 reversed** (Computation 3.1): **nearly one in three.**
6. **0 on one side, 0.295238 on the other** (Observation 3.4): **same representation.**
7. **(4, 9) gives 1.000000** (Computation 4.1): **coprime.**
8. **(4, 6) gives 0.500000** (Computation 4.1): **only half occur.**
9. **(6, 10, 15) gives 0.033333** (Computation 4.1): **the lcm decides.**
10. **arithmetic, or order** (Observation 5.1): **the separator; Papers 605, 300.**

Closing: Split into remainders, addition and multiplication survive but order vanishes. With moduli 3, 5 and 7 the 105 numbers give the 105 triples exactly once each, and both addition and multiplication work componentwise with no discrepancy at all, because both operations close inside the remainders. Yet for order, only 1260 of 10920 pairs (0.115385) have all three remainders aligned, and 372 of those (0.295238) run against the true order, because order rides not on any one remainder but on the position the three together fix. Without coprimality fewer tuples occur at all, 0.500000 for (4, 6) and 0.033333 for (6, 10, 15), decided by the least common multiple rather than the product. Placed among the earlier papers: beside the seat where Paper 605 counted how many combinations occur, and treating arithmetic and order as distinct roots by the criterion of Paper 300. To be honest: only the moduli 3, 5, 7 and a few small sets were counted; the general case was not.

References

1. Hardy, G. H., & Wright, E. M. (2008). *An Introduction to the Theory of Numbers*. Oxford University Press.
2. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. YOI: <https://www.yamagishiyuuki.com/yoi/300>; DOI: 10.5281/zenodo.22171914

3. Yamagishi, Y. (2026). *The Sexagenary Cycle Has Sixty Pairs, Not a Hundred and Twenty* [Paper 605]. YOI:
<https://www.yamagishiyuuki.com/yoi/605>; DOI: 10.5281/zenodo.23014440

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)