

“The Same Shape” Does Not Mean One Thing: The Count Changes With Whether Reflections Are Identified

— Counting shapes made of n squares gives, for $n = 8$, **2725** when nothing is moved, **704** when rotations are identified, and **369** when reflections are too — the separator is whether reflections are identified or only rotations — [Paper 847]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/847>

— *From thinking to syncing.* —

Abstract

“How many shapes are there” has something to settle before counting begins. This paper counts that there are three ways to settle it, and three answers. No new theorem or law is claimed. First, one n yields three answers: at $n = 8$: fixed **2725**, rotations only **704**, free **369** (Section 2). Second, identifying reflections alone moves the count a long way: **704** against **369** at $n = 8$, a difference of **335** (Section 2). Third, and this is the core. The ratio of fixed to free never reaches 8. It stands at **7.384824** at $n = 8$ (Section 3). Fourth, the shortfall is due to symmetric shapes: **53** of the **369** at $n = 8$ have fewer than 8 placements under rotation and reflection (Section 3). Fifth, this count closes from the other side too: summing each free shape’s placements reproduces the fixed count at every n from 1 to 8 (Section 3). Sixth, the separator is whether reflections are identified or only rotations: the count is not a property of the shapes but a consequence of what is called the same (Section 4). **Keywords:** polyomino, enumeration, symmetry, orbit, identification, reflection.

Scope of this paper (scope note): No new theorem or law is claimed: that polyominoes are counted in fixed, one-sided and free forms, that the resulting sequences are known, and that symmetric shapes shorten their orbits (Burnside's counting), are all known. Real puzzles are not discussed: how to pack them and how to play with them are not treated. Tiling is not treated: whether a rectangle can be filled is not the subject. No formula for the counts is claimed: no closed form for the number of polyominoes is known, and none is offered; only enumerated results are laid out. The size is not generalised: $n = 9$ and beyond were not counted. Three dimensions are not entered: solid shapes are not treated. Relation to earlier papers: Paper 182 showed that "the curse of dimensionality" is not one curse. This paper does not sit in that seat but stands beside it: there one name pointed at several difficulties, here one question admits three answers. Paper 300 showed that whether two things share a root is decidable: the three ways of counting are distinct roots by that criterion. What is added is laying the three counts side by side from $n = 1$ to 8 by one enumeration, tying the shortfall below 8 to the number of symmetric shapes, confirming as a separate computation that the orbit sizes sum to the fixed count throughout, and placing the separator on whether reflections are identified.

1. Notation and the Question

Setting: consider shapes made by joining unit squares along edges. "The same shape" is settled in three ways.

fixed : translation only, one-sided : and rotation, free : and reflection

There are 4 rotations and 8 placements once reflections are allowed, so for a shape with no symmetry the fixed count should be 8 times the free count.

Question: is "how many" one number?

2. Three Answers

Computation 2.1: from $n = 1$ to 8, all three counts were taken.

n	fixed	rotations only	free
1	1	1	1
2	2	1	1
3	6	2	2
4	19	7	5
5	63	18	12
6	216	60	35
7	760	196	108
8	2725	704	369

Observation 2.2

three answers stand under one n . At $n = 8$: 2725, 704, 369: nothing about the shapes changed; only the rule for “the same” did.

Observation 2.3

reflections alone move it by 335. That is the gap between 704 and 369 at $n = 8$: at $n = 4$ it is 7 against 5, and at $n = 3$ both are 2: at small sizes there is no gap at all.

Observation 2.4

the fixed sequence 1, 2, 6, 19, 63, 216, 760, 2725 and the free sequence 1, 1, 2, 5, 12, 35, 108, 369 match the known ones. That is a check that the enumeration here runs correctly: had they not matched, the first thing to doubt would be the counting.

3. The Ratio Never Reaches 8

Computation 3.1: the ratio of fixed to free, and the number of symmetric shapes, were laid out.

n	fixed / free	symmetric shapes
4	3.800000	4
5	5.250000	7
6	6.171429	15
7	7.037037	24
8	7.384824	53

Observation 3.2

this is the core. The ratio approaches 8 without reaching it. At $n = 8$ it is 7.384824: a symmetric shape has fewer than 8 placements under rotation and reflection, and the fixed count falls short by exactly that much.

Observation 3.3

symmetric shapes do not die out. Their number rises with n : 4, 7, 15, 24, 53: the ratio approaches 8 because asymmetric shapes grow faster, not because symmetric ones vanish.

Computation 3.4: a separate computation: for each free shape, sum the number of placements it admits.

n	sum of placements	fixed count
5	63	63
6	216	216
7	760	760
8	2725	2725

Observation 3.5

they match at every n from 1 to 8. One counts fixed shapes directly; the other sums placements over free shapes, so “short by the symmetric ones” is not a phrase but an identity among numbers.

4. The Separator

Observation 4.1

the separator is whether reflections are identified or only rotations. A shape cut from paper can be turned over; one printed on its face cannot, which to take is a property not of the shapes but of the things being handled.

Observation 4.2

so “how many” is not one number. At $n = 8$, each of 2725, 704 and 369 is a correct answer, which one to give must be settled before the question is asked.

Observation 4.3

placed beside Paper 182.182 showed that “the curse of dimensionality” is not one curse. There one name pointed at several difficulties; here one question admits three answers. The difference is whether the multiplicity sits in the name or in the question. Paper 300 showed that whether two things share a root is decidable: the three counts are distinct roots by that criterion.

Observation 4.4

what is not said. Real puzzles, tiling, formulas for the counts and three dimensions are not treated. What is shown is that a count is a consequence of what one calls the same.

5. Census and Closing

1. **fixed 2725 at $n = 8$** (Computation 2.1): **translation only.**
2. **rotations only: 704** (Computation 2.1): **no turning over.**
3. **free: 369** (Computation 2.1): **reflections identified.**
4. **a gap of 335** (Observation 2.3): **from reflections alone.**
5. **ratio 7.384824** (Computation 3.1): **the core; short of 8.**
6. **53 symmetric shapes** (Computation 3.1): **the reason for the shortfall.**
7. **symmetric shapes grow too** (Observation 3.3): **they do not vanish.**
8. **placements sum to fixed** (Computation 3.4): **at every n from 1 to 8.**
9. **all three are correct** (Observation 4.2): **settle it before asking.**
10. **are reflections the same** (Observation 4.1): **the separator; Papers 182, 300.**

Closing: “The same shape” does not mean one thing: the count changes with whether reflections are identified. Shapes made of eight squares number 2725 when nothing is moved, 704 when rotations are identified and 369 when reflections are too: nothing about the shapes changed; only the rule for “the same” did. The ratio of fixed to free approaches 8 without reaching it, standing at 7.384824 at $n = 8$, because a symmetric shape has fewer than 8 placements, and 53 of the 369 at $n = 8$ are of that kind. That this is exact, and not a manner of speaking, is closed by a separate computation: summing each free shape’s placements reproduces the fixed count at every n from 1 to 8. Placed among the earlier papers: beside Paper 182’s “one name, several difficulties”, in the form of one question with three answers, and treating the three counts as distinct roots by the criterion of Paper 300. To be honest: only up to $n = 8$ was counted, and neither larger sizes nor three dimensions were.

References

-
1. Golomb, S. W. (1994). *Polyominoes: Puzzles, Patterns, Problems, and Packings*. Princeton University Press.
 2. Yamagishi, Y. (2026). “*The Curse of Dimensionality*” *Is Not One Curse* [Paper 182]. YOI: <https://www.yamagishiyuuki.com/yoi/182>; DOI: 10.5281/zenodo.22104170
 3. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. YOI: <https://www.yamagishiyuuki.com/yoi/300>; DOI: 10.5281/zenodo.22171914
-

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)