

An Index That Divides Weight by the Square of Height Moves With Height Even at the Same Build

— Keeping the build fixed and changing only height makes mass grow as the cube of height, so an index dividing by the square moves as the first power — the separator is whether one asks about shape or about weight

— [Paper 848]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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Abstract

Weight divided by the square of height is widely used as a measure of build. This paper counts how that “square” departs from the logic of similarity. No new theorem or law is claimed. First, at the same build, mass grows as the cube of height: scaling lengths by k scales volume by k^3 (Section 2). Second, and this is the core. An index dividing by the square moves even at the same build: from 1.5 m to 1.9 m it grows by a factor of **1.266667** (Section 2). Third, raising the power shrinks the movement: **1.125463** for the 2.5th power, and 1.000000, no movement, for the cube (Section 2). Fourth, the same build crosses a cut-off: a build scoring 22.0 at 1.5 m scores **27.866667** at 1.9 m, above 25 (Section 3). Fifth, keeping the value means changing the build: to stay at 22.0 at 1.9 m, mass must be **0.789474** of the similar mass, the reciprocal of the height ratio (Section 3). Sixth, the separator is whether one asks about shape or about weight: the square-law index mixes shape and size into one number (Section 4). **Keywords:** BMI, body-build index, similarity, power law, height, model.

Scope of this paper (scope note): No new theorem or law is claimed: that weight divided by height squared (the Quetelet index, BMI) is in use, that mass is proportional to the cube of length under similarity, and that an index dividing by the cube (the Rohrer or ponderal index) also exists, are all known. No medical advice is given, which values are healthy and which cut-offs to use are not stated. No distribution of real people is claimed: people do not necessarily grow similarly; similarity is a model placed for comparison. No index is chosen: whether to use the square or the cube is not stated, and the population fits behind the square are not entered. The body's composition is not entered: fat and muscle fractions are not treated. Relation to earlier papers: Paper 256 showed that the only scale at which dynamic similarity is exact is one. This paper stands beside it: there changing scale breaks similarity; here keeping similarity while changing scale makes the square-law index move. Paper 300 showed that whether two things share a root is decidable: shape and weight are distinct roots by that criterion. What is added is laying out the movement at a fixed build for powers 2, 2.5 and 3, counting a case where the same build crosses a cut-off, confirming that the mass factor needed to hold the value is the reciprocal of the height ratio, and placing the separator on shape versus weight.

1. Notation and the Question

Setting: let M be mass and h height, and consider the index formed by dividing mass by height to the power q . Keeping the build and changing only size, mass is proportional to the cube of height.

$$I_q = \frac{M}{h^q}, \quad M \propto h^3 \implies I_q \propto h^{3-q}$$

The widely used choice is $q = 2$.

Question: lining up people of the same build at different heights, how does the index move?

2. Movement at the Same Build

Computation 2.1: from 1.5 m to 1.9 m (a ratio of 1.266667), the factor on the index was computed for each power.

power q	factor on the index at the same build
2.0	1.266667
2.5	1.125463
3.0	1.000000

Observation 2.2

this is the core. Dividing by the square, the index moves even at the same build. The factor 1.266667 is the height ratio itself: the first power left over from the cube minus the square stays in the index.

Observation 2.3

only dividing by the cube keeps it still. At the 2.5th power it still moves by 1.125463: for an index to see build alone, its power must match the power of mass.

3. Crossing a Cut-Off

Computation 3.1: a build with a square-law index of 22.0 at 1.5 m was stretched unchanged to 1.9 m.

quantity	value
square-law index at 1.5 m	22.0
square-law index at 1.9 m	27.866667
above the cut-off at 25	yes
factor on the cube-law index	1.000000

Observation 3.2

the same build crosses the cut-off on the square-law index only. The cube-law index stays where it was. What crossed the line was not the build but the height.

Computation 3.3: conversely, the mass needed to hold the square-law index at 22.0 at 1.9 m was found.

quantity	value
factor on the similar mass	0.789474
reciprocal of the height ratio	0.789474

Observation 3.4

holding the value requires a slimmer build. The factor equals the reciprocal of the height ratio exactly: when a tall person and a short person share a value, the tall person has the slimmer build.

4. The Separator

Observation 4.1

the separator is whether one asks about shape or about weight. The cube-law index sees shape alone; the square-law index mixes shape and size into one number: not which is right, but what each number is looking at.

Observation 4.2

placed beside Paper 256.256 showed that the only scale at which dynamic similarity is exact is one. There, changing scale breaks similarity; here, keeping similarity while changing scale makes the square-law index move. Whether the object breaks or the number does. Paper 300 showed that whether two things share a root is decidable: shape and weight are distinct roots by that criterion.

Observation 4.3

what is not said. Which values are healthy, which index to use and whether people actually grow similarly are not treated. What is shown is that the square-law index moves as the first power of height at a fixed build.

5. Census and Closing

1. same build, mass as the cube (Section 1): because volume.
2. square: factor **1.266667** (Computation 2.1): the core.
3. that is the height ratio itself (Observation 2.2): a first power remains.
4. 2.5th power: **1.125463** (Computation 2.1): still moves.

5. **cube: 1.000000** (Computation 2.1): **does not move**.
6. **22.0 becomes 27.866667** (Computation 3.1): **at the same build**.
7. **crosses the cut-off at 25** (Computation 3.1): **height crossed it**.
8. **holding it needs 0.789474** (Computation 3.3): **reciprocal height ratio**.
9. **equal values, taller is slimmer** (Observation 3.4): **different shapes**.
10. **shape, or weight** (Observation 4.1): **the separator; Papers 256, 300**.

Closing: An index that divides weight by the square of height moves with height even at the same build. Keeping the build and changing height from 1.5 m to 1.9 m, the square-law index grows by 1.266667, the 2.5th-power index by 1.125463, and only the cube-law index stays at 1.000000, because mass grows as the cube of height, and dividing by the square leaves a first power behind. So a build scoring 22.0 at 1.5 m scores 27.866667 at 1.9 m and crosses the cut-off at 25, while holding 22.0 at 1.9 m requires slimming to 0.789474 of the similar mass. What crossed the line was height, not build. The square-law index mixes shape and size into one number; the cube-law index sees shape alone. Placed among the earlier papers: beside Paper 256's "similarity breaks with scale", as the side where keeping similarity makes the number move, and treating shape and weight as distinct roots by the criterion of Paper 300. To be honest: people do not necessarily grow similarly; similarity here is a model placed for comparison.

References

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