

Pitch Sees Only the Ratio of Tension to Weight, but the Instrument Bears the Tension Itself

— Producing the same 110 Hz on a 0.648 m steel string takes 125.3011 N at 1.0 mm diameter and 31.3253 N at 0.5 mm, a ratio of 4.000000 — the separator is whether one asks about pitch or about the load on the instrument

— [Paper 857]

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Abstract

Tightening a string raises its pitch, thickening it lowers it. This paper counts how the two fold into one ratio, and what that ratio leaves for the instrument to bear. No new theorem or law is claimed. First, the same pitch needs very different tensions at different thicknesses: 110 Hz needs 125.3011 N at 1.0 mm and 31.3253 N at 0.5 mm (Section 2). Second, the ratio is the square of the thickness: doubling the diameter needs 4.000000 times the tension (Section 2). Third, and this is the core. Pitch sees only the ratio: doubling both tension and linear density keeps 110 Hz, while the load on the instrument alone doubles to 250.6022 N (Section 3). Fourth, raising pitch needs tension as the square: 1.122462 times for a semitone and 4 times for an octave (Section 3). Fifth, a slight change in tension puts the pitch out: a 1 per cent change gives 8.613176 cents (Section 3). Sixth, the separator is whether one asks about pitch or about the load on the instrument: two strings sounding the same note can load an instrument four times differently (Section 4). **Keywords:** string tension, linear density, Mersenne's laws, cent, load on the instrument, model.

Scope of this paper (scope note): No new theorem or law is claimed: that a string's fundamental is $\frac{1}{2L} \sqrt{T/\mu}$ (Mersenne's laws), that linear density is proportional to cross-sectional area, that an octave needs four times the tension, and the definition of the cent, are all known. No real instrument is claimed: a length of 0.648 m, steel density 7850 kg/m³ and diameters of 1.0 mm and 0.5 mm are model values matching no instrument or brand of string. Wound strings are not included: only plain steel strings are treated. Bending stiffness is not included: the departure of overtones from ideal caused by stiffness is not treated. Music is not discussed: timbre, playing and tuning systems are not the subject. Relation to earlier papers: Paper 466 showed that the impossibility of tuning everything at once is a proved fact, and its scope note left instruments open. This paper stands beside it: there the side of pitch ratios, here the side of the string that makes a pitch. Paper 300 showed that whether two things share a root is decidable: pitch and the load on the instrument are distinct roots by that criterion. What is added is sounding one pitch on two thicknesses and comparing tensions, measuring that scaling tension and linear density together leaves pitch unchanged but changes the load, measuring that 1 per cent of tension is 8.613176 cents, and placing the separator on pitch versus load.

1. Notation and the Question

Setting: consider a string of length L , tension T and linear density μ . For a plain steel string, linear density is density times cross-sectional area.

$$f = \frac{1}{2L} \sqrt{\frac{T}{\mu}}, \quad \mu = \rho \frac{\pi d^2}{4}, \quad T = (2Lf)^2 \mu$$

Only the ratio T/μ enters the pitch f . Here $L = 0.648$ m and $\rho = 7850$ kg/m³.

Question: do strings sounding the same pitch load the instrument equally?

2. Same Pitch, Different Tension

Computation 2.1: the same 110 Hz was produced on two thicknesses.

diameter	linear density	tension needed
1.0 mm	6.165376×10^{-3} kg/m	125.3011 N
0.5 mm	1.541344×10^{-3} kg/m	31.3253 N
ratio	—	4.000000

Observation 2.2

at the same pitch, tension differs as the square of the thickness. Doubling the diameter quadruples linear density, so holding the pitch quadruples the tension: fed back into the formula, both give 110 Hz to within 10^{-9} .

3. The Ratio, and What It Leaves Out

Computation 3.1: on the 1.0 mm string, tension and linear density were both doubled.

quantity	original string	both doubled
pitch	110 Hz	110.000000 Hz
load on the instrument (tension)	125.3011 N	250.6022 N

Observation 3.2

this is the core. Pitch sees only the ratio. Pitch stays at 110.000000 Hz while the load on the instrument doubles to 250.6022 N: in the pitch formula T and μ fold into a quotient, while the load is T as it stands.

Computation 3.3: the tension factors needed to change pitch, and the pitch error from a tension change, were computed.

quantity	value
tension factor for a semitone	1.122462
tension factor for an octave	4.000000
error from a 1 per cent tension change	8.613176 cents

Observation 3.4

pitch moves only as the square root of tension. Hence an octave needs four times the tension, and 1 per cent of tension is just under 0.5 per cent of pitch, 8.613176 cents: the load on the instrument, by contrast, moves in proportion to the tension.

4. The Separator

Observation 4.1

the separator is whether one asks about pitch or about the load on the instrument. The first is set by the ratio T/μ , the second by T itself: two strings sounding the same note can load an instrument four times differently.

Observation 4.2

placed beside Paper 466.466 showed that the impossibility of tuning everything at once is a proved fact, and left instruments open. There the side where pitch ratios cannot all be matched, here the side of the string that makes a pitch, where a quantity absent from the pitch remains on the instrument. Paper 300 showed that whether two things share a root is decidable: pitch and load are distinct roots by that criterion.

Observation 4.3

what is not said. Real instruments, wound strings, stiffness-shifted overtones and timbre are not treated. What is shown is that outside the ratio folded into the pitch, a load remains for the instrument.

5. Census and Closing

1. pitch is the ratio T/μ (Section 1): folded by division.
2. 1.0 mm needs **125.3011** N (Computation 2.1): **110** Hz.
3. 0.5 mm needs **31.3253** N (Computation 2.1): **the same 110** Hz.
4. ratio **4.000000** (Computation 2.1): **the square of thickness**.

5. **both doubled, still 110.000000 Hz** (Computation 3.1): **the core.**
6. **load doubles to 250.6022 N** (Computation 3.1): **outside the ratio.**
7. **semitone 1.122462** (Computation 3.3): **tension goes as the square.**
8. **octave 4** (Computation 3.3): **likewise.**
9. **1 per cent tension, 8.613176 cents** (Computation 3.3): **slight but audible.**
10. **pitch, or load** (Observation 4.1): **the separator; Papers 466, 300.**

Closing: Pitch sees only the ratio of tension to weight, but the instrument bears the tension itself. Producing 110 Hz on a 0.648 m steel string takes 125.3011 N at 1.0 mm and 31.3253 N at 0.5 mm, a ratio of 4.000000: doubling the diameter quadruples linear density, and holding the pitch quadruples the tension. Doubling both tension and linear density keeps the pitch at 110.000000 Hz while the load on the instrument doubles to 250.6022 N: the pitch formula folds T and μ into a quotient, while the load is T as it stands. Pitch moves only as the square root of tension, so a semitone needs 1.122462 times, an octave 4 times, and a 1 per cent change in tension is 8.613176 cents. Placed among the earlier papers: beside Paper 466's side where pitch ratios cannot all be matched, as the side of the string that makes a pitch, and treating pitch and load as distinct roots by the criterion of Paper 300. To be honest. This is a plain-steel model, without wound strings or stiffness-shifted overtones.

References

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