

Whether Book Pages Cockle Depends Less on How Damp the Paper Gets Than on Which Direction Runs Along the Bound Edge

— the direction along the spine is bound, so the free edge cockles, with an amplitude of **0.349083** mm when the grain runs parallel to the spine and **0.700507** mm when it runs across — the separator is whether one asks how damp the paper gets or which direction is constrained — [Paper 859]

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Abstract

Bookbinding says “run the grain parallel to the spine”. This paper counts what that alignment actually aligns. No new theorem or law is claimed. First, paper expands differently in different directions: a 30 per-cent humidity rise gives 0.003 along the grain and 0.012 across it (Section 2). Second, the direction along the bound edge is constrained: the expansion with nowhere to go becomes a wave along the free edge (Section 2). Third, and this is the core. Which direction is constrained changes the wave height: amplitude **0.349083** mm with grain parallel to the spine, **0.700507** mm across it (Section 3). Fourth, the waves differ less than the strains: fourfold strain gives amplitude only **2.006707** times (Section 3). Fifth, amplitude goes as the square root of strain: the approximation $(w/\pi)\sqrt{\epsilon}$ agrees with the exact arc-length solution within 1 per cent (Section 3). Sixth, the separator is whether one asks how damp the paper gets or which direction is constrained: the same paper, equally damp, cockles differently according to orientation (Section 4). **Keywords:** paper grain, hygroexpansion, anisotropy, cockling, bookbinding, model.

Scope of this paper (scope note): No new theorem or law is claimed: that paper’s hygroexpansion depends on fibre direction and is larger across the grain, the bookbinding practice of running the grain parallel to the spine, that a thin constrained sheet buckles into waves, and the relation between a sinusoid’s arc length and amplitude, are all known.No real paper values are claimed: expansion coefficients of 1×10^{-4} and 4×10^{-4} and a 20 mm wavelength are model values.No binding instructions are given: procedures and choice of materials are not stated.Drying is not included: curl on drying and shrinking is not treated.Paper strength is not treated: tearing and creasing are not the subject.Relation to earlier papers: Paper 342 showed that mixing the same two materials in the same proportion lets stiffness differ by 16.93 with arrangement. This paper stands beside it: there arrangement changes stiffness, here orientation changes cockling.Paper 300 showed that whether two things share a root is decidable: how damp and which direction is constrained are distinct roots by that criterion.What is added is solving the wave amplitude from the exact arc length for both orientations, measuring that fourfold strain gives 2.006707 times the amplitude, confirming that the approximation agrees within 1 per cent, and placing the separator on dampness versus constrained direction.

1. Notation and the Question

Setting: when paper takes up $\Delta\phi$ of relative humidity, it strains $\beta_{MD}\Delta\phi$ along the grain and $\beta_{CD}\Delta\phi$ across it.The direction along the spine is bound and cannot extend; the constrained strain ε is taken to appear as a sinusoid of wavelength w along the free edge.

$$\frac{1}{w} \int_0^w \sqrt{1 + \left(\frac{2\pi a}{w} \cos \frac{2\pi x}{w} \right)^2} dx = 1 + \varepsilon, \quad a \approx \frac{w}{\pi} \sqrt{\varepsilon}$$

The left equation is solved for the amplitude a ; the right is the small-strain approximation.Here $\beta_{MD} = 1 \times 10^{-4}$, $\beta_{CD} = 4 \times 10^{-4}$, $\Delta\phi = 30$ and $w = 20$ mm.

Question: is cockling set by how damp the paper gets or by how its directions are placed?

2. Direction-Dependent Expansion

Computation 2.1: the strains for a 30 per-cent humidity rise were computed.

direction	strain
along the grain	0.003000
across the grain	0.012000
ratio	4.000000

Observation 2.2

the same humidity gives fourfold different expansion by direction. Which direction runs along the spine decides the constrained strain: the dampness is the same; only the constrained direction differs.

3. Wave Height

Computation 3.1: the free-edge amplitude was solved from the exact arc-length equation.

grain relative to spine	constrained strain	amplitude (exact)	amplitude (approx.)
parallel	0.003000	0.349083 mm	0.348691 mm
across	0.012000	0.700507 mm	0.697382 mm

Observation 3.2

this is the core. Orientation changes the wave height. With the grain across the spine the edge cockles 2.006707 times as high as with it parallel: aligning grain with spine means putting the small-expansion direction on the bound edge.

Observation 3.3

fourfold strain gives only twofold amplitude. The ratio is 2.006707, close to the square root of 4: excess length is absorbed as the square of the wave height, so height goes as the square root of strain.

Observation 3.4

approximation and exact solution agree within 1 per cent. The exact amplitude is slightly larger because of higher terms in the arc length: the same reason puts the ratio slightly above 2.

4. The Separator

Observation 4.1

the separator is whether one asks how damp the paper gets or which direction is constrained. At equal dampness, the constrained direction splits the amplitude into 0.349083 mm and 0.700507 mm: the same paper, equally damp, cockles differently according to orientation.

Observation 4.2

placed beside Paper 342.342 showed that the same two materials in the same proportion differ in stiffness by 16.93 with arrangement. There arrangement changes stiffness; here orientation changes cockling. In both, direction and placement alone change the answer without changing the contents. Paper 300 showed that whether two things share a root is decidable: dampness and constrained direction are distinct roots by that criterion.

Observation 4.3

what is not said. Real paper coefficients, binding procedures, curl on drying and paper strength are not treated. What is shown is that cockling is set by the constrained direction.

5. Census and Closing

1. along the grain **0.003000** (Computation 2.1): **small**.
2. across **0.012000** (Computation 2.1): **four times**.
3. the spine direction is **bound** (Section 1): **the excess becomes a wave**.
4. parallel: **0.349083** mm (Computation 3.1): **a low wave**.
5. across: **0.700507** mm (Computation 3.1): **the core**.
6. ratio **2.006707** (Observation 3.3): **not four**.
7. height as the square root of strain (Observation 3.3): **absorbed as a square**.
8. approximation within **1 per cent** (Observation 3.4): **higher terms**.
9. what is aligned is the small expansion (Observation 3.2): **not etiquette**.

10. dampness, or constrained direction (Observation 4.1): the separator; Papers 342, 300.

Closing: Whether book pages cockle depends less on how damp the paper gets than on which direction runs along the bound edge. A 30 per-cent humidity rise expands paper 0.003 along the grain and 0.012 across it, and because the spine direction is bound, its share becomes a wave along the free edge: the constrained expansion has nowhere else to go. With a 20 mm wavelength the amplitude is 0.349083 mm with the grain parallel to the spine and 0.700507 mm across it, so fourfold strain yields only 2.006707 times the height: excess length is absorbed as the square of the wave height, so height goes as the square root of strain. Running the grain with the spine means putting the small-expansion direction on the bound edge. Placed among the earlier papers: beside Paper 342's "arrangement changes stiffness", as "orientation changes cockling", and treating dampness and constrained direction as distinct roots by the criterion of Paper 300. To be honest: the coefficients and wavelength are model values, and curl on drying is not included.

References

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