

Destroying Half the Dye Does Not Make the Cloth Half White

— Starting from an absorbance of **2** (transmitting **0.01**) and destroying the dye at a constant fractional rate, half the dye remaining still lets through only **0.100000** of the light, far short of **0.5** — the separator is whether one asks how much dye remains or how much light gets through — [Paper 861]

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September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/861>

— *From thinking to syncing.* —

Abstract

Bleaching is described as destroying the dye. This paper counts how the destruction of dye and the return of whiteness fall out of step. No new theorem or law is claimed. First, with half the dye left, little light gets through: transmission rises from 0.010000 only to 0.100000 (Section 2). Second, whiteness arrives near the end: 0.630957 at a tenth of the dye left and 0.794328 at a twentieth (Section 2). Third, and this is the core. The cloth looks half white long after the dye is half gone: the ratio of times is **2.732021** (Section 3). Fourth, the ratio is set by how dark the start is. It is $\log_2 A_0 + 1.732021$, so darker means a longer wait (Section 3). Fifth, half the light is $\log_{10} 2$ in absorbance: at unit absorbance the dye left at half transmission is **0.301030**, meeting the shared-root group of Papers 288, 612 and 719 as a fourth (Section 4). Sixth, the separator is whether one asks how much dye remains or how much light gets through: the first falls at a constant fractional rate; the second returns as its exponential (Section 5). **Keywords:** bleaching, absorbance, Beer-Lambert law, first-order reaction, light transmitted, model.

Scope of this paper (scope note): No new theorem or law is claimed: that transmitted light depends exponentially on absorbance (the Beer--Lambert law), that absorbance is proportional to the amount of dye, and that first-order reactions decay exponentially, are all known. No real bleaching is claimed: an initial absorbance of 2 and first-order destruction are model choices matching no agent or fibre. Reaction mechanisms are not entered, which bonds break and how is not treated. Fibre damage is not treated: weakening of the cloth is not the subject. Perception is not included: the logarithmic sense of brightness is not included; only the fraction of light transmitted is examined. Relation to earlier papers: Paper 719 showed that colour mixing is not one operation, one side adding and the other multiplying transmittances, placing the Beer--Lambert law among the known. This paper does not sit in that seat but stands beside it: there colours are layered, here dye is destroyed. Papers 288, 612 and 719 grouped $\log_{10} 2 = 0.301030$ as "the share of a decade taken by a factor of 2", a shared root. Here the same number appears as the change in absorbance that halves the light. Since it comes from the same source, that half the light is $\log_{10} 2$ in common logarithm, it is called a shared root in the sense of Paper 300. What is added is laying out dye remaining against light transmitted, measuring the time ratio 2.732021 , confirming that the ratio is $\log_2 A_0 + 1.732021$ and separating 1.732021 from $\sqrt{3}$, and placing the separator on dye amount versus light transmitted.

1. Notation and the Question

Setting: let c be the fraction of dye remaining, A_0 the initial absorbance and T the fraction of light transmitted. Dye is destroyed at a constant fractional rate (first order).

$$T = 10^{-A_0 c}, \quad c(t) = e^{-kt}, \quad \frac{t_{T=1/2}}{t_{c=1/2}} = \log_2 A_0 - \log_2(\log_{10} 2)$$

The right-hand expression follows because $T = 1/2$ occurs at $c = \log_{10} 2 / A_0$. Here $A_0 = 2$ and $k = 1$.

Question: when half the dye is destroyed, is the cloth half white?

2. Dye Remaining and Light Transmitted

Computation 2.1: the fraction of light transmitted was computed for each fraction of dye remaining.

dye remaining	light transmitted
1.00	0.010000
0.50	0.100000
0.25	0.316228
0.10	0.630957
0.05	0.794328
0.00	1.000000

Observation 2.2

with half the dye left, little light gets through. Transmission is 0.100000, far from 0.5: light depends exponentially on the amount of dye, so while it is dark, destruction changes the look very little.

Observation 2.3

whiteness arrives near the end. As the remaining dye goes from 0.1 to 0, transmission climbs from 0.630957 to 1: much of the visible change happens while the last tenth is destroyed.

3. The Lag in Time

Computation 3.1: with rate constant 1, the times to half dye and to half light were computed.

quantity	value
time to half the dye	0.693147
time to half the light	1.893693
dye remaining then	0.150515
ratio of times	2.732021

Observation 3.2

this is the core. Looking half white comes much later. Only after 2.732021 times the dye's half-time does the light reach half: by then only 0.150515 of the dye remains.

Computation 3.3: the initial absorbance was varied and the time ratio computed.

initial absorbance	dye left at half light	ratio of times
0.5	0.602060	0.732021
1.0	0.301030	1.732021
2.0	0.150515	2.732021
3.0	0.100343	3.316983

Observation 3.4

the ratio is $\log_2 A_0 + 1.732021$. All four agree within 10^{-12} , and each doubling of absorbance adds 1 to the ratio: the darker the start, the longer before anything looks different.

Observation 3.5

a misleading closeness is cut away. The constant 1.732021 is $-\log_2(\log_{10} 2) = 1.732020846$, differing from $\sqrt{3} = 1.732050808$ by 2.996×10^{-5} : $\sqrt{3}$ appears in Paper 860 of the same batch, from a different source.

4. Half the Light Is $\log_{10} 2$

Observation 4.1

at unit absorbance, the dye left at half transmission is $\log_{10} 2$ itself. The value 0.301029996 matches $\log_{10} 2 = 0.301029996$: halving the light means lowering absorbance by exactly $\log_{10} 2$.

Observation 4.2

this is the fourth member of a shared root. Paper 719 grouped 0.301030, with Paper 288 (digits gained per bisection) and Paper 612 (the share of leading digit 1), as the share of a decade taken by a factor of 2. Here the same number is the width within a decade of absorbance taken by a factor of 2 in light. The source is the same, so by the sense of Paper 300 it is a shared root.

5. The Separator

Observation 5.1

the separator is whether one asks how much dye remains or how much light gets through. The first falls at a constant fractional rate; the second returns as its exponential: “how far bleached” points at times 2.732021 apart depending on which is measured.

Observation 5.2

placed beside Paper 719. 719 showed that one side of colour mixing multiplies transmittances. There colours are layered, here dye is destroyed. Layering darkens by multiplication; destroying brightens, late, by an exponential.

Observation 5.3

what is not said. Real bleaches and fibres, reaction mechanisms, fibre damage and perception are not treated. What is shown is that the fall of dye and the return of whiteness are out of step.

6. Census and Closing

1. half the dye, light 0.100000 (Computation 2.1): **not half.**
2. a tenth left, 0.630957 (Computation 2.1): **white comes late.**
3. dye half-time 0.693147 (Computation 3.1): **the first mark.**
4. light half-time 1.893693 (Computation 3.1): **much later.**

5. ratio **2.732021** (Computation 3.1): **the core**.
6. ratio $\log_2 A_0 + 1.732021$ (Observation 3.4): **darker waits longer**.
7. **1.732021** is not $\sqrt{3}$ (Observation 3.5): **difference 2.996×10^{-5}** .
8. at unit absorbance, **0.301030** (Observation 4.1): **$\log_{10} 2$ itself**.
9. a **fourth shared root** (Observation 4.2): **Papers 288, 612, 719**.
10. **dye amount, or light** (Observation 5.1): **the separator; Papers 719, 300**.

Closing: Destroying half the dye does not make the cloth half white. From an absorbance of 2, half the dye remaining still transmits only 0.100000, and the light reaches half only 2.732021 times as long after the dye does, when just 0.150515 of the dye is left: transmission depends exponentially on the dye, so while it is dark, destruction barely changes the look. The ratio is $\log_2 A_0 + 1.732021$ in the initial absorbance, and the constant 1.732021 is distinct from $\sqrt{3}$ by 2.996×10^{-5} : the ratio comes from half the light being $\log_{10} 2$ in common logarithm. At unit absorbance the dye left at half light is exactly 0.301030, meeting the shared-root group of Papers 288, 612 and 719 as a fourth. Placed among the earlier papers: beside Paper 719's "layering multiplies", as "destroying returns late by an exponential", and calling $\log_{10} 2$ a shared root in the sense of Paper 300. To be honest: an absorbance of 2 and first-order destruction are model choices, and perceived brightness is not included.

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