

The Higher the Recycling Rate, the More the Last Point Counts

— If a fraction r of material returns for another use each time, one unit of new material serves $1/(1 - r)$ uses: **2** at $r = 0.5$, **10** at **0.9** and **100** at **0.99** — the separator is whether one asks about the share that returns or the share that does not — [Paper 862]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/862>

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Abstract

Recycling rates are quoted as “how much came back”. This paper counts that new material is set by “how much did not”. No new theorem or law is claimed. First, the number of uses is the reciprocal of the share that does not return: **10** at $r = 0.9$, **20** at **0.95**, **100** at **0.99** (Section 2). Second, and this is the core. The same improvement counts for more at a higher rate: from **0.90** to **0.95** new material falls to **0.500000**; from **0.50** to **0.55**, only to **0.900000** (Section 3). Third, the rate is a product of two shares: collection **0.9** and yield **0.8** give **0.72** and **3.571429** uses (Section 4). Fourth, which one is raised matters: collection to **1.0** gives **5.000000** uses; yield to **0.9** gives **5.263158** (Section 4). Fifth, the formula matches summing use by use: $1/(1 - r)$ and the running sum of quantities shrinking by r each round agree within 10^{-9} (Section 2). Sixth, the separator is whether one asks about the share that returns or the share that does not: new material is proportional to the latter, so each point weighs more as the rate rises (Section 5). **Keywords:** recycling rate, geometric series, collection, yield, new material, model.

Scope of this paper (scope note): No new theorem or law is claimed: the sum of a geometric series $1/(1 - r)$, that the recycling rate is the product of collection rate and process yield, and that improvements count for more at higher rates, are all known. No real values are claimed: r , collection 0.9 and yield 0.8 are model values matching no scheme or statistic. Merits are not discussed: environmental effect and cost are not treated. Degradation is not included: limits on the number of uses from material wearing with each cycle are not included. Definitions are not entered: counting rates by weight or by item is not treated. Relation to earlier papers: Paper 446 showed that the theoretical floor rises when you ask for more. This paper stands beside it: there the last point of a high recovery rate is costly; here the last point of a high rate is effective. The cost side and the effect side of the same last point. Paper 519 showed that Zeno's four paradoxes do not share a single resolution, placing the sum of a geometric series among the known in its scope note. Here that sum appears directly as the number of uses. What is added is laying out uses and new material by rate, measuring that the same 0.05 splits into 0.500000 and 0.900000, measuring how the number of uses depends on raising collection or yield, and placing the separator on returned share versus lost share.

1. Notation and the Question

Setting: each time material is used, a fraction r returns for another use and $1 - r$ is lost. The number of uses U per unit of new material, and the new material per use, are as follows.

$$U = \sum_{n=0}^{\infty} r^n = \frac{1}{1 - r}, \quad \text{new material per use} = 1 - r, \quad r = c \times y$$

c is the collection share and y the process yield.

Question: when the rate rises, how much less new material is needed?

2. Number of Uses

Computation 2.1: uses and new material per use were computed for each rate.

r	uses	new material per use
0.50	2.000000	0.500000
0.55	2.222222	0.450000
0.70	3.333333	0.300000
0.90	10.000000	0.100000
0.95	20.000000	0.050000
0.99	100.000000	0.010000

Observation 2.2

uses are the reciprocal of the lost share. As $1 - r$ goes from 0.1 to 0.01, uses go from 10 to 100: the formula and the running sum of quantities shrinking by r each round agree within 10^{-9} .

3. Same Improvement, Different Effect

Computation 3.1: the same improvement of 0.05 was compared at a low and a high rate.

improvement	factor on new material
0.50 to 0.55	0.900000
0.90 to 0.95	0.500000

Observation 3.2

this is the core. The higher rate gains far more. From 0.90 to 0.95 new material halves; from 0.50 to 0.55 it falls by only a tenth: new material follows the lost share, and 0.10 to 0.05 is a halving while 0.50 to 0.45 is a tenth.

4. Collection and Yield

Computation 4.1: taking the rate as collection times yield, raising each was compared.

case	rate	uses
collection 0.9 , yield 0.8	0.720000	3.571429
collection raised to 1.0	0.800000	5.000000
yield raised to 0.9	0.810000	5.263158

Observation 4.2

raising each by 0.1 does not give the same result. Yield at 0.9 gives 5.263158 uses, more than the 5.000000 from collection at 1.0: in a product, raising the smaller factor raises the rate more.

5. The Separator

Observation 5.1

the separator is whether one asks about the share that returns or the share that does not. Rates of 0.90 and 0.95 differ by only 0.05 in the returned share but by a factor of 2 in the lost share: new material is proportional to the latter, so each point weighs more as the rate rises.

Observation 5.2

placed beside Papers 446 and 519. 446 showed that the theoretical floor rises when you ask for more. There the last point of a high recovery is costly; here the last point is effective. The cost side and the effect side of the same last point. 519 placed the geometric-series sum among the known. Here that sum appears directly as the number of uses.

Observation 5.3

what is not said. Real schemes and statistics, environmental effect and cost, degradation and differing definitions are not treated. What is shown is that new material is proportional to the share that does not return.

6. Census and Closing

1. uses are $1/(1 - r)$ (Section 1): **a geometric series.**
2. **10 uses at 0.9** (Computation 2.1): **lost share 0.1.**
3. **100 at 0.99** (Computation 2.1): **0.01.**
4. **0.90 to 0.95: 0.500000** (Computation 3.1): **the core.**
5. **0.50 to 0.55: 0.900000** (Computation 3.1): **only a tenth less.**
6. **rate is collection times yield** (Computation 4.1): **0.720000.**
7. **collection to 1.0: 5.000000 uses** (Computation 4.1): **one way up.**
8. **yield to 0.9: 5.263158 uses** (Computation 4.1): **counts for more.**
9. **formula and sum within 10^{-9}** (Observation 2.2): **independent.**
10. **returned share, or lost share** (Observation 5.1): **the separator; Papers 446, 519.**

Closing: The higher the recycling rate, the more the last point counts. If a fraction r returns each time, one unit of new material serves $1/(1 - r)$ uses, 10 at 0.9 and 100 at 0.99, because new material follows not the share that returns but the share that does not. So the same 0.05 improvement multiplies new material by 0.500000 from 0.90 to 0.95 but only by 0.900000 from 0.50 to 0.55: the lost share halves in one case and shrinks by a tenth in the other. Writing the rate as collection 0.9 times yield 0.8, collection at 1.0 gives 5.000000 uses and yield at 0.9 gives 5.263158, so raising the smaller factor counts for more. Placed among the earlier papers: beside Paper 446's "the last point of a high rate is costly", as "the last point of a high rate is effective", and counting the geometric sum Paper 519 placed among the known as the number of uses. To be honest: rates, collection and yield are model values, and limits from material degradation are not included.

References

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