

The Time One Molecule Spends in the Air Is Not the Time an Added Excess Takes to Vanish

— If a share **0.25** leaves the air each year and returns from a partner reservoir that holds **3** times the air's amount at equilibrium, one molecule stays in the air for **4.000000** years on average — the separator is whether one asks about molecules coming and going or about where the excess can go — [Paper 865]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Abstract

If molecules are replaced within a few years, it is easy to assume an added amount also vanishes within a few years. This paper counts, in a model of two linked reservoirs, that the time a molecule stays and the time an excess takes to vanish are different quantities. No new theorem or law is claimed. First, one molecule stays in the air for **4.000000** years on average: if a share **0.25** leaves each year, that is the air's amount divided by the outgoing flow (Section 1). Second, and this is the core. An added excess stays far longer than molecular residence suggests: after **20** years the closed pair of reservoirs still holds **0.250954**, which is **37.244946** times the 6.737947×10^{-3} expected from molecular residence alone (Section 2). Third, the part above equilibrium falls faster than molecular residence, but then the fall stops: the excess approaches the equilibrium share **0.250000** with a time constant of **3.000000** years and is still **0.250000** after 100 years. Once the partner fills, excess that left comes back (Section 3). Fourth, even with an exit, the last decline is slow: if a share **0.002** leaves the partner each year, the excess after 100 years is **0.217171** and the remaining decline has a time constant of **667.671180** years, **166.917795** times the molecular residence (Section 3). Fifth, the numbers were obtained two ways and matched: Runge--Kutta integration matched the exact solution for the closed pair and the eigenvalue solution with the exit, all within 10^{-13} (Sections 2 and 3). Sixth, the separator is whether one asks about molecules coming and going or about where the excess can go: molecules coming and going are set by the outgoing share alone; where the excess can go is set by the partner's capacity and its exit (Section 4). **Keywords:** residence time, box model, equilibrium, time constant, eigenvalue, model.

Scope of this paper (scope note): No new theorem or law is claimed: reservoirs linked by first-order rates, residence time as amount divided by flow, and the difference between molecular residence time and the time for an added excess to return, are all distinctions known in atmospheric chemistry. No real values are claimed: the outgoing share 0.25, the partner's size 3 and the exit 0.002 are model values matching no gas, atmosphere or ocean. No chemistry is included: loss by reaction and changes in temperature or solubility are not included. No reservoirs are added: the partner is one reservoir, and mixing within it is not treated. Climate is not discussed. What effects the excess has, or how it should be reduced, is not treated. Relation to earlier papers: Paper 549 showed that the time inside changes almost sixfold while "number inside = arrival rate $\times$ time inside" does not move. This paper stands beside it. The molecular residence of 4.000000 years follows from that relation, but the decline of an added excess does not; the partner reservoir sets it. What is added is laying out molecular residence and the remaining excess in the same model, measuring that after 20 years 37.244946 times the expected excess remains, obtaining the slow time constant of 667.671180 years from eigenvalues and matching it to integration, and placing the separator on molecules coming and going versus where the excess can go.

1. Notation and the Question

Setting: let A be the excess in the air and O the excess in the partner reservoir. With k_1 the share leaving the air, k_2 the share returning from the partner, and k_s an exit from the partner, they are linked as follows.

$$\frac{dA}{dt} = -k_1 A + k_2 O, \quad \frac{dO}{dt} = k_1 A - (k_2 + k_s) O, \quad \tau_{\text{molecule}} = \frac{1}{k_1}$$

We set $k_1 = 0.25$ per year and $k_2 = k_1/3$ (at equilibrium the partner holds 3 times the air's amount), with two cases for the exit: $k_s = 0$ (closed) and $k_s = 0.002$ per year. An excess of 1 was added to the air, with 0 in the partner. Numbers were obtained by Runge--Kutta integration (step 0.001 years, 248000 steps in total).

Question: if molecules are replaced in 4 years, does an added excess also vanish in about 4 years?

2. Molecular Residence and the Remaining Excess

Computation 2.1: the excess left in the air was compared for the closed pair, the pair with an exit, and the value $e^{-k_1 t}$ expected from molecular residence alone.

years	closed pair	with exit	molecular residence only
4	0.447698	0.447460	3.678794×10^{-1}
20	0.250954	0.245758	6.737947×10^{-3}
100	0.250000	0.217171	1.388794×10^{-11}

Observation 2.2

this is the core. An added excess stays far longer than molecular residence suggests. After 20 years the closed pair holds 0.250954, which is 37.244946 times the 6.737947×10^{-3} expected from molecular residence alone: molecules are replaced in 4 years, but part of the excess that left comes back from the partner.

Observation 2.3

the integration matches the exact solution. The closed pair's excess is $1/4 + (3/4)e^{-(k_1+k_2)t}$, and it differed from the integration by less than 10^{-13} at all three times: the total of the two reservoirs stayed at 1.000000.

3. Equilibrium and Time Constants

Computation 3.1: the closed pair's equilibrium and time constant, and the two time constants with the exit, were computed.

quantity	value
molecular residence $1/k_1$	4.000000 years
closed pair: share left in air at equilibrium	0.250000
closed pair: time constant $1/(k_1 + k_2)$	3.000000 years
with exit: fast time constant	2.995486 years
with exit: slow time constant	667.671180 years
slow time constant $\div$ molecular residence	166.917795
weight of the slow part	0.252260

Observation 3.2

the part above equilibrium falls faster than molecular residence, but then the fall stops. The time constant of 3.000000 years is shorter than the molecular residence of 4.000000 years: but only the part above equilibrium declines, and the closed pair's excess never drops below 0.250000. The slow eigenvalue becomes 0 when the exit is set to 0, so the excess does not vanish.

Observation 3.3

even with an exit, the last decline is slow. If a share 0.002 leaves the partner each year, the part of the excess with weight 0.252260 declines with a time constant of 667.671180 years: 166.917795 times the molecular residence. The eigenvalue solution differed from the integration by less than 2×10^{-15} at all three times, and the excess after 100 years was 0.217171.

4. The Separator

Observation 4.1

the separator is whether one asks about molecules coming and going or about where the excess can go. Molecules coming and going are set only by the share k_1 leaving the air, where the excess can go is set by how much the partner can hold (k_2) and how much leaves it (k_s). Exchange replaces molecules; it does not remove the excess.

Observation 4.2

placed beside Paper 549.549 showed that “number inside = arrival rate $\times$ time inside” does not move while the time inside changes almost sixfold: the molecular residence of 4.000000 years is the air’s amount divided by the outgoing flow under that relation. The decline of an added excess does not follow from it; the partner reservoir, with its returning flow, sets it.

Observation 4.3

what is not said. Real gases, atmospheres and oceans, chemical reactions, temperature changes, and the effects or reduction of the excess are not treated. What is shown is that, in one and the same model, the time a molecule stays and the time an added excess takes to vanish are different quantities.

5. Census and Closing

1. molecular residence $1/k_1 = 4.000000$ years (Computation 3.1): **amount over flow.**
2. excess after 20 years **0.250954** (Computation 2.1): **closed pair.**
3. **37.244946** times the expectation (Observation 2.2): **the core.**
4. time constant **3.000000** years (Computation 3.1): **shorter than molecular residence.**
5. never below the equilibrium **0.250000** (Observation 3.2): **it comes back.**
6. with exit, **0.217171** after 100 years (Computation 2.1): **exit 0.002.**
7. slow time constant **667.671180** years (Computation 3.1): **from eigenvalues.**
8. **166.917795** times molecular residence (Observation 3.3): **the last decline.**

9. integration and solutions within 10^{-13} (Observations 2.3, 3.3): independent.

10. molecules coming and going, or where the excess can go (Observation 4.1): the separator; Paper 549.

Closing: The time one molecule spends in the air is not the time an added excess takes to vanish. If a share 0.25 leaves the air each year, one molecule stays 4.000000 years on average, yet an added excess still stands at 0.250954 after 20 years, 37.244946 times what molecular residence alone would leave, because part of the excess that left comes back from the filled partner. The excess approaches the equilibrium share 0.250000 with a time constant of 3.000000 years and, without an exit, falls no further: with an exit of 0.002, the remaining decline has a time constant of 667.671180 years, 166.917795 times the molecular residence. Placed among the earlier papers: beside the molecular residence that follows from Paper 549's "number inside = arrival rate $\times$ time inside", we placed the decline of an excess that does not follow from it. To be honest: rates and reservoir sizes are model values, and chemical reactions, temperature changes and mixing within reservoirs are not included.

References

1. Jacob, D. J. (1999). *Introduction to Atmospheric Chemistry*. Princeton University Press.
2. Seinfeld, J. H., & Pandis, S. N. (2006). *Atmospheric Chemistry and Physics: From Air Pollution to Climate Change* (2nd ed.). Wiley.
3. Yamagishi, Y. (2026). *The Time Inside Changes Almost Sixfold, and the Product Does Not Move* [Paper 549]. YOI: <https://www.yamagishiyuuki.com/yoi/549>; DOI: 10.5281/zenodo.23014185

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