

The Folding Order Moves Pages on the Sheet but Not Which Pages Share a Side or the Sum 17 Across the Last Fold

— sums of adjacent panels meeting at the first or second fold differ from order to order, but pairs meeting at the last fold sum only to **17** in all three — the separator is whether one asks about position on the sheet or about which side and which fold brings pages together — [Paper 877]

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— *From thinking to syncing.* —

Abstract

An imposition diagram looks like page numbers scattered at random. This paper tries all three orders of folding one sheet three times and counts what in the arrangement changes with the folding and what does not. No new theorem or law is claimed. First, the arrangement on the sheet changes with the folding: the front top row reads 15, 2, 3, 14 for V--H--V, 15, 10, 11, 14 for V--V--H and 15, 2, 7, 10 for H--V--V; every two of the three differ (Section 2). Second, which pages share a side does not change: in all three, the front carries {2, 3, 6, 7, 10, 11, 14, 15} and the back {1, 4, 5, 8, 9, 12, 13, 16} (Section 2). Third, back-to-back pages are an odd number and the next in all: the front and back of the same panel formed pairs such as 1 and 2, 3 and 4, in all three orders (Section 2). Fourth, and this is the core. Adjacent pages across the last fold sum only to 17 in all three: sorting adjacent panels by the fold at which they first meet, every pair meeting at the third (last) fold summed to 17 in every order (Section 3). Fifth, sums for pairs meeting at earlier folds change with the folding: pairs meeting at the first fold summed to {5, 13}, {13, 21} and {5, 13, 21, 29}, and at the second to {9, 25}, {9, 25} and {9} (Section 3). Sixth, the separator is whether one asks about position on the sheet or about which side and which fold brings pages together: positions swap with the folding order, but in all three orders the side sets and the sum across the last fold were the same (Section 4).

Keywords: imposition, signature, page numbering, folding order, spine.

Scope of this paper (scope note): No new theorem or law is claimed: that a sheet is folded into a signature, that the imposition arrangement changes with the folding, and that back-to-back pages and pages across a fold follow sum rules, are all known in printing and binding. No procedures are prescribed: how plates are laid out on real presses and what binders do are not stated. Size is not widened: only 16-page signatures from 4×2 panels folded three times are counted; 32 pages or more, bleed and gripper margins are not treated. One reading is fixed: pages are numbered from the top of the folded stack, reading the last fold as the spine. Relation to earlier papers: Paper 859 showed that whether book pages cockle depends less on how damp the paper gets than on which direction runs along the bound edge. This paper stands beside it. There the direction of the paper along the bound edge was counted. Here we count which panel of one sheet comes to which side and which side of which fold. What is added is laying out the sheet arrangement for all three folding orders, checking that the set of pages on the front is the same in all three, sorting adjacent sums by the fold at which pairs meet, and placing the separator on position versus side and order of meeting.

1. Notation and the Question

Setting: consider a sheet with 4 panels across and 2 down on its front (side A) and back (side B). “Vertical” folds left onto right along a vertical line, “horizontal” folds top onto bottom along a horizontal line, and three folds make one stack. From the top of the stack, the upward face gets an odd number and the downward face the next; the sheet is then opened and the page read panel by panel.

$$V \rightarrow H \rightarrow V, \quad V \rightarrow V \rightarrow H, \quad H \rightarrow V \rightarrow V$$

A horizontal fold can be made only once, so these three orders are all there are. The last fold is read as the spine. For each pair of adjacent panels, the fold at which they first come to the same position was recorded.

Question: in the imposition arrangement, what changes with the folding and what does not?

2. Three Arrangements

Computation 2.1: for the three folding orders, the page on each panel of the sheet was found.

folding order	side	top row	bottom row
V--H--V	A	15, 2, 3, 14	10, 7, 6, 11
V--H--V	B	16, 1, 4, 13	9, 8, 5, 12
V--V--H	A	15, 10, 11, 14	2, 7, 6, 3
V--V--H	B	16, 9, 12, 13	1, 8, 5, 4
H--V--V	A	15, 2, 7, 10	14, 3, 6, 11
H--V--V	B	16, 1, 8, 9	13, 4, 5, 12

Observation 2.2

the arrangement changes, but which pages share a side does not. Every two of the three orders give different arrangements on the sheet. Yet side A carries $\{2, 3, 6, 7, 10, 11, 14, 15\}$ in all three, pages leaving remainder 2 or 3 on division by 4. In every order, 1 to 16 each appeared exactly once.

Observation 2.3

back-to-back pages are an odd number and the next in all. The A and B pages of the same panel formed $(1, 2), (3, 4), \dots, (15, 16)$ in all three orders, because one panel of the sheet becomes the front and back of one leaf in the stack.

3. Sums of Adjacent Pages

Computation 3.1: sums of pages on adjacent panels were sorted by the fold at which the pair first meets (both sides A and B).

folding order	meet at first fold	meet at second fold	meet at third fold
V--H--V	$\{5, 13\}$	$\{9, 25\}$	$\{17\}$
V--V--H	$\{13, 21\}$	$\{9, 25\}$	$\{17\}$
H--V--V	$\{5, 13, 21, 29\}$	$\{9\}$	$\{17\}$

Observation 3.2

this is the core. Adjacent pages across the last fold sum only to 17 in all three orders. Pairs meeting at the third fold are left--right neighbours in V--H--V and H--V--V and top--bottom neighbours in V--V--H, a different direction on the sheet. Yet all of them sum to 17. At the last fold the upper and lower halves of the stack meet as mirror images, so the j -th face from the top meets the j -th face from the bottom.

Observation 3.3

sums for pairs meeting at earlier folds change with the folding. Pairs meeting at the first fold summed to $\{5, 13\}$, $\{13, 21\}$ and $\{5, 13, 21, 29\}$, and at the second to $\{9, 25\}$, $\{9, 25\}$ and $\{9\}$: the roadmap's "paired pages always sum to 17" holds for pairs across the last fold, not for all neighbours.

4. The Separator

Observation 4.1

the separator is whether one asks about position on the sheet or about which side and which fold brings pages together. Position on the sheet changes merely by swapping the folding order, which side a page lands on and at which fold it meets its neighbour are set by the order within the numbered stack of the signature. In all three folding orders, the side sets fixed by that order and the sum 17 across the last fold were the same.

Observation 4.2

placed beside Paper 859.859 showed that whether pages cockle depends on which direction runs along the bound edge. There, the direction of the paper along the bound edge was counted. Here we counted which panel of one sheet comes to which side and which side of which fold. Both ask which part of the paper ends up at the spine.

Observation 4.3

what is not said. Real plate layouts or binding procedures, signatures of 32 pages or more, and bleed or gripper margins are not treated. What is shown is that the arrangement changes with the folding while the side sets and the sum across the last fold do not.

5. Census and Closing

1. three folds make 16 pages (Section 1): only three orders.
2. top rows 15, 2, 3, 14, 15, 10, 11, 14, 15, 2, 7, 10 (Computation 2.1): the arrangement changes.
3. 1 to 16 once each (Observation 2.2): in all three.
4. side A is {2, 3, 6, 7, 10, 11, 14, 15} (Observation 2.2): side sets do not change.
5. remainders 2 or 3 (Observation 2.2): on division by 4.
6. back to back: odd and next (Observation 2.3): front and back of one leaf.
7. sum 17 at the last fold (Observation 3.2): the core.
8. first-fold sums differ (Observation 3.3): up to {5, 13, 21, 29}.
9. second-fold sums differ too (Observation 3.3): {9, 25} and {9}.
10. position, or side and order of meeting (Observation 4.1): the separator; Paper 859.

Closing: Changing the folding order changes where pages sit on the sheet, but not which pages share a side or the sum 17 across the last fold. In the three orders of folding three times, the front top row reads 15, 2, 3, 14, 15, 10, 11, 14 and 15, 2, 7, 10, yet the front carries {2, 3, 6, 7, 10, 11, 14, 15} in all three, and back-to-back pages are an odd number and the next, because side and order of meeting are set not by position on the sheet but by the order within the numbered stack. Adjacent sums for pairs meeting at the first or second fold differ from order to order, but pairs meeting at the last fold sum only to 17 in all three, because at the last fold the two halves of the stack meet as mirror images. Placed among the earlier papers: beside Paper 859's "the direction along the bound edge", we placed which part of one sheet comes to which side and which fold. To be honest: only 16-page signatures from 4×2 panels folded three times were counted, reading the last fold as the spine.

References

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