

Rebound Is Restitution Times Relative Speed, So Lowering the Cup at Half the Ball's Speed Equals Halving Restitution

— A ball dropped from 0.3 m (2.426108 m/s) caught on a cup with restitution 0.526621 rebounds at 1.277638 m/s relative to a still cup — the separator is whether one changes the cup's material or how the cup moves —

[Paper 882]

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Abstract

It is easy to assume that landing a ball quietly on a kendama cup requires a soft cup. This paper counts, for a cup moving while it catches, that rebound is the product of restitution and relative speed. No new theorem or law is claimed. First, on a still cup, rebound is restitution times impact speed: a ball striking at 2.426108 m/s after falling 0.3 m, caught with restitution 0.526621, rebounds at 1.277638 m/s (Section 2). Second, and this is the core. Lowering the cup at half the ball's speed equals halving restitution: the rebound seen from the cup is 0.638819 m/s, the same as keeping the cup still with restitution 0.263310 (Section 2). Third, lowering it at the ball's speed gives zero rebound: relative speed is 0, so there is no impact, whatever the cup's material (Section 2). Fourth, following the contact in steps, under the formula's assumptions the cup's speed does not matter: with a spring and damper, no gravity and pulling forces allowed, the ratio was 0.526626 at all four cup speeds (Section 3). Fifth, dropping those assumptions makes restitution itself move: cutting off pulling forces alone raises the ratio uniformly to 0.571747, independent of cup speed; adding gravity alone lowers it from 0.523486 to 0.513842 as relative speed falls (Section 3). Sixth, the separator is whether one changes the cup's material or how the cup moves: material changes restitution, motion changes relative speed, and as a product they act on rebound equally (Section 4). **Keywords:** coefficient of restitution, relative speed, kendama, spring-damper contact, rebound.

Scope of this paper (scope note): No new theorem or law is claimed: that restitution is a ratio of relative speeds, that a spring--damper contact has restitution $\exp(-\zeta\pi/\sqrt{1-\zeta^2})$, and that restitution moves with the contact assumptions, are all known. No real values are claimed: drop height, spring stiffness and damping ratio are model values matching no kendama. Spin and string are not included: ball spin, string tension and cup shape are not included; the cup moves at constant speed. No technique is recommended: how one should catch is not stated. Relation to earlier papers: Paper 355 showed that the coefficient of restitution is not a material constant, and that the empirical law points at its own breaking speed. This paper stands beside it. There restitution was counted as moving with impact speed. Here we count that relative speed can be changed by how the cup moves, and that in a stepped contact restitution moves with relative speed once gravity is included. What is added is laying out rebound for each cup-lowering speed, counting that lowering at half speed equals halving restitution, matching the stepped contact under the formula's assumptions and separating the deviations by dropping one assumption at a time, and placing the separator on material versus motion.

1. Notation and the Question

Setting: a ball falling downward at speed v is caught by a cup moving downward at constant speed u . Restitution e is the ratio of the speed of separation to the speed of approach, seen from the cup.

$$v_{\text{sep}} - u = e(v - u), \quad e = \exp\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)$$

The latter is the restitution of a spring--damper contact with damping ratio ζ , giving 0.526621 at $\zeta = 0.2$. The ball falls from 0.3 m, striking at 2.426108 m/s. The contact was followed in steps of 10^{-7} seconds with stiffness 5×10^4 N/m and ball mass 0.08 kg.

Question: is a softer cup the only way to reduce rebound?

2. Cup-Lowering Speed and Rebound

Computation 2.1: with the cup lowered at 0, 0.25, 0.5, 0.75 and 1 times the impact speed, the rebound was computed.

share of impact speed	cup speed (m/s)	rebound seen from cup (m/s)	upward speed seen from ground (m/s)
0	0.000000	1.277638	1.277638
0.25	0.606527	0.958229	0.351702
0.5	1.213054	0.638819	−0.574235
0.75	1.819581	0.319410	−1.500171
1	2.426108	0.000000	−2.426108

Observation 2.2

this is the core. Lowering the cup at half the ball's speed equals halving restitution. Lowering the cup at half speed gives a rebound of 0.638819 m/s seen from the cup: getting this with a still cup needs restitution 0.263310, exactly half of 0.526621. Rebound is the product of restitution and relative speed, so halving either gives the same result.

Observation 2.3

lowering the cup at the ball's speed gives zero rebound. When the cup's speed equals the impact speed, relative speed is 0 and there is no rebound, whatever the material: seen from the ground, once the cup is lowered at half the speed or more, the ball is still moving downward after the catch.

3. Following the Contact in Steps

Computation 3.1: following a spring--damper contact in steps, the ratio seen from the cup was computed with and without gravity and pulling forces.

assumptions	share 0	0.25	0.5	0.75
no gravity, pulling allowed	0.526626	0.526626	0.526626	0.526626
no gravity, no pulling	0.571747	0.571747	0.571747	0.571747
gravity, pulling allowed	0.523486	0.522430	0.520312	0.513842
gravity, no pulling	0.568512	0.567425	0.565238	0.558572

Observation 3.2

under the formula's assumptions, the stepped ratio does not depend on cup speed. With no gravity and pulling allowed, the ratio was 0.526626 at all four speeds, matching the formula's 0.526621: seen from a cup moving at constant speed, the contact looks the same as with a still cup.

Observation 3.3

dropping the assumptions makes restitution itself move. Cutting off pulling forces alone raises the ratio uniformly to 0.571747, whatever the cup's speed: adding gravity alone lowers it from 0.523486 to 0.513842 as the cup speeds up and relative speed falls. The first run used only gravity with no pulling and did not match the formula; dropping one assumption at a time separated the uniform rise from the speed dependence.

4. The Separator

Observation 4.1

the separator is whether one changes the cup's material or how the cup moves. Changing material changes restitution; changing motion changes relative speed: rebound is their product, so both act equally. Without a softer cup, lowering it along with the ball reduces the rebound.

Observation 4.2

placed beside Paper 355.355 showed that restitution is not a material constant and moves with impact speed. Here too, in a stepped contact, restitution was measured moving with relative speed once gravity was included. On top of that, we counted that relative speed itself can be changed by how the cup moves.

Observation 4.3

what is not said. Real kendama material values, ball spin, string tension and catching technique are not treated. What is shown is that rebound is the product of restitution and relative speed, and lowering the cup at half speed equals halving restitution.

5. Census and Closing

1. $\text{rebound} = e(v - u)$ (Section 1): seen from the cup.
2. 1.277638 m/s on a still cup (Computation 2.1): $e = 0.526621$.
3. 0.638819 m/s lowered at half speed (Computation 2.1): the core.
4. restitution 0.263310 for the same rebound (Observation 2.2): exactly half.
5. 0 lowered at the ball's speed (Observation 2.3): material does not matter.
6. 0.526626 under the formula's assumptions (Computation 3.1): independent of cup speed.
7. 0.571747 without pulling (Observation 3.3): a uniform rise.
8. 0.523486 to 0.513842 with gravity (Observation 3.3): moves with relative speed.
9. one assumption dropped at a time (Observation 3.3): causes measured, not guessed.
10. material, or motion (Observation 4.1): the separator; Paper 355.

Closing: Rebound is the product of restitution and relative speed, so lowering the cup at half the ball's speed equals halving the coefficient of restitution. A ball dropped from 0.3 m onto a cup with restitution 0.526621 rebounds at 1.277638 m/s from a still cup and 0.638819 m/s from a cup lowered at half speed, the latter equal to restitution 0.263310, because rebound is a product, halving restitution by material or halving relative speed by motion acts equally. Following the contact with a spring and damper, the ratio is 0.526626 whatever the cup's speed under the formula's assumptions, but 0.571747 uniformly without pulling forces, and lower at smaller relative speed with gravity: restitution moves with relative speed under some contact assumptions. Placed among the earlier papers:

beside Paper 355’s “restitution is not a material constant”, we placed changing relative speed by how the cup moves. To be honest: spring stiffness and damping are model values, and ball spin and string are not included.

References

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