

The Efficiency of Boiling Water Is Set by the Ratio of Power to Heat Loss, Not the Appliance, and Weaker Heat Wastes More

— in a vessel with $h = 3$ W/K, efficiency is **0.497068** at 300 W and **0.959444** at 3000 W, so the weaker heat uses **1.930207** times the energy — the separator is whether one asks about the kind of appliance or about power divided by the heat-loss coefficient — [Paper 883]

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Abstract

It is easy to assume that the efficiency of boiling water is set by whether one uses a kettle or a pan. This paper counts, in a simple model of heating while heat escapes, that efficiency is set by the ratio of power to heat loss. No new theorem or law is claimed. First, in the same vessel, weaker heat is less efficient: with $h = 3$ W/K, efficiency is **0.497068** at 300 W, **0.874518** at 1000 W and **0.959444** at 3000 W (Section 2). Second, weaker heat uses nearly twice the energy for the same water: 300 W uses **1.930207** times the energy of 3000 W, because heat keeps escaping during the long time to boil (Section 2). Third, and this is the core. If power divided by the heat-loss coefficient is the same, different vessels have the same efficiency: $h = 1$ at 1000 W and $h = 3$ at 3000 W both give **0.959444**; $h = 3$ at 500 W and $h = 6$ at 1000 W both give **0.734027**; all four pairs agree within 10^{-12} (Section 3). Fourth, at or below $80h$ the water never boils: for a vessel with $h = 6$ W/K the threshold is 480 W, and at 300 W the water never reaches 100 °C (Section 2). Fifth, the formula's times match stepped times: for every case, the formula and time followed in steps agreed within 0.02 seconds (Section 2). Sixth, the separator is whether one asks about the kind of appliance or about power divided by the heat-loss coefficient: a kettle looks more efficient than a pan not because of what it is but because that ratio is larger (Section 4). **Keywords:** boiling water, efficiency, heat loss, Newton's law of cooling, power.

Scope of this paper (scope note): No new theorem or law is claimed: the heat to warm water, Newton's law of cooling, and the temperature of a body heated while cooling are all standard in heat transfer. No real products are claimed: power and heat-loss coefficients are model values matching no kettle and no pan. Combustion and evaporation are not included: the share of a gas flame's heat reaching the pan, lids, heat carried off by steam, and the vessel's own heat capacity are not included. Nothing is recommended, which appliance to use is not stated. Relation to earlier papers: Paper 468 showed that the same proportional increase buys less at the top. This paper stands beside it. There, efficiency gains from raising a ratio were counted as dulling at the high end. Here too, efficiency rises with power but the rise dulls, and what sets it is the ratio of power to heat loss. What is added is laying out time, energy and efficiency for three coefficients and five powers, measuring that weaker heat uses 1.930207 times the energy, checking that equal ratios give equal efficiency in different vessels, and placing the separator on the kind of appliance versus the ratio.

1. Notation and the Question

Setting: 1 kg of water (heat capacity $C = 4186$ J/K) is heated from 20°C to 100°C at power P . Heat escapes at h W/K in proportion to how far the water temperature T is above the surroundings (20°C).

$$C \frac{dT}{dt} = P - h(T - 20), \quad t_{\text{boil}} = \frac{C}{h} \ln \frac{P}{P - 80h}, \quad \text{efficiency} = \frac{C \times 80}{P t_{\text{boil}}}$$

If $P \leq 80h$, the water never boils. Times were obtained from the formula and by following the heating with Runge--Kutta steps of 0.01 seconds.

Question: what sets the efficiency of boiling water?

2. Power and Efficiency

Computation 2.1: with $h = 3$ W/K and varying power, time to boil, energy used and efficiency were computed.

power (W)	time (s)	energy (kWh)	efficiency
300	2245.702367	0.187142	0.497068
500	912.445398	0.126729	0.734027
1000	382.930879	0.106370	0.874518
2000	178.370164	0.099095	0.938722
3000	116.345138	0.096954	0.959444

Observation 2.2

in the same vessel, weaker heat is less efficient. At 300 W efficiency is 0.497068, about half the 0.959444 at 3000 W: 300 W uses 1.930207 times the energy of 3000 W. The water takes in the same 0.093022 kWh, but heat keeps escaping during the 2245.702367 seconds to boil.

Observation 2.3

at or below $80h$ the water never boils. For a vessel with $h = 6$ W/K the threshold is 480 W, and at 300 W it never reaches 100 °C: heat in and heat out balance below 100 °C. For every case, the formula and stepped times agreed within 0.02 seconds.

3. When the Ratio Is the Same

Computation 3.1: pairs of different vessels with the same power divided by heat-loss coefficient, P/h , were compared.

P/h (K)	first vessel	second vessel	efficiency
166.7	$h = 3, 500$ W	$h = 6, 1000$ W	0.734027
333.3	$h = 3, 1000$ W	$h = 6, 2000$ W	0.874518
500.0	$h = 1, 500$ W	$h = 6, 3000$ W	0.917676
1000.0	$h = 1, 1000$ W	$h = 3, 3000$ W	0.959444

Observation 3.2

this is the core. With the same ratio, different vessels have the same efficiency. In all four pairs, the two efficiencies agreed within 10^{-12} : in the efficiency formula, P and h appear only as P/h , so the heat taken in by the water, $C \times 80$, and the share lost are set by that ratio alone.

4. The Separator

Observation 4.1

the separator is whether one asks about the kind of appliance or about power divided by the heat-loss coefficient. Asking about the kind of appliance gives the answer that a kettle beats a pan: asking about the ratio, a kettle looks better because its power is high and heat escapes little; a well-insulated pan on strong heat reaches the same efficiency.

Observation 4.2

placed beside Paper 468.468 showed that the same proportional increase buys less at the top. Here too, tripling P/h from 333.3 to 1000 raises efficiency only from 0.874518 to 0.959444. The higher it goes, the more the gain dulls.

Observation 4.3

what is not said. Real product efficiencies, the share of a flame's heat reaching a pan, heat carried off by steam, and which appliance to use are not treated. What is shown is that efficiency is set by the ratio of power to heat loss, and weaker heat wastes more.

5. Census and Closing

1. heat into water **0.093022 kWh** (Section 1): **1 kg by 80 K**.
2. efficiency **0.497068** at **300 W** (Computation 2.1): **weak heat**.
3. **0.959444** at **3000 W** (Computation 2.1): **strong heat**.
4. energy **1.930207** times (Observation 2.2): **the loss of weak heat**.
5. no boiling at or below **80h** (Observation 2.3): **480 W for $h = 6$** .
6. same ratio, same efficiency (Observation 3.2): **the core**.
7. **$h = 3$ at 500 W and $h = 6$ at 1000 W** (Computation 3.1): **both 0.734027**.
8. tripling the ratio: **0.874518 to 0.959444** (Observation 4.2): **the gain dulls**.
9. formula and stepped times agree (Observation 2.3): **within 0.02 s**.

10. kind of appliance, or ratio (Observation 4.1): the separator; Paper 468.

Closing: The efficiency of boiling water is set not by the kind of appliance but by the ratio of power to heat loss, and weaker heat wastes more. In a vessel with $h = 3 \text{ W/K}$, efficiency is 0.497068 at 300 W and 0.959444 at 3000 W, and the weaker heat uses 1.930207 times the energy, because heat keeps escaping during the long time to boil. With the same power divided by the heat-loss coefficient, different vessels agree in efficiency within 10^{-12} , and at or below $80h$ the water never boils: a kettle looks better not because of what it is, but because that ratio is larger. Placed among the earlier papers: beside Paper 468's "the same increase buys less at the top", we placed that boiling efficiency is set by a ratio whose gains dull as it rises. To be honest: power and heat-loss coefficients are model values, and neither the transfer of a flame's heat to a pan nor steam is included.

References

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