

What Makes a Hairspring Clock Drift with Temperature Is Not Expansion but the Change in Stiffness

— the stiffness contribution is **20.000000** times the net of the two expansions, which cancel each other — the separator is whether one asks about the dimensions of the wheel and spring or about the stiffness of the material

— [Paper 885]

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Abstract

It is easy to assume that clocks drift with temperature because metal expands and contracts. This paper separates and counts three contributions by which temperature moves the period of a balance wheel and hairspring. No new theorem or law is claimed. First, the daily error is the sum of three contributions: in an all-steel model it is **−9.849600** seconds a day per K: **−1.036800** from the wheel's expansion, **−10.368000** from the change in stiffness and **+1.555200** from the spring's expansion (Section 2). Second, and this is the core. The main actor is the change in stiffness: the two expansions act in opposite directions and cancel, leaving a net **0.518400** seconds. The stiffness contribution is **20.000000** times that (Section 2). Third, a brass wheel barely changes the error: a steel spring with a brass wheel gives **−10.454400** seconds a day; the difference in expansion is small beside the stiffness contribution (Section 2). Fourth, choosing the stiffness coefficient removes the error: with a brass wheel and a spring expanding at 0.8×10^{-5} , a stiffness coefficient of $+1.4 \times 10^{-5}$ brings the daily error to 0 (Section 3). Fifth, the formula matches recomputed periods: changing dimensions and stiffness by 0.001 K and recomputing the period gave slopes agreeing with the formula within 1.9×10^{-6} seconds a day for all three cases (Section 2). Sixth, the separator is whether one asks about the dimensions of the wheel and spring or about the stiffness of the material: changes in dimension cancel and leave little, while the change in stiffness has nothing to cancel it (Section 4).

Keywords: hairspring, balance wheel, temperature coefficient, elastic modulus, clock drift.

Scope of this paper (scope note): No new theorem or law is claimed: the period formula for a balance wheel and hairspring, spring stiffness set by the elastic modulus and dimensions, and that the temperature change of stiffness is the main cause of drift, countered by special alloys, are all known.No real values are claimed: expansion and stiffness coefficients are model values matching no clock and no alloy.Other causes are not included: magnetism, position error, oil viscosity and the shape of the spring’s coils are not included.Nothing is designed, which material to use is not stated.Relation to earlier papers: Paper 856 showed that what throws a pendulum off when it is pushed is where it is pushed, more than how hard. This paper stands beside it. There the escapement’s push was counted as upsetting the period. Here the three temperature contributions are separated, counting that the main actor is stiffness, not dimensions.What is added is laying out the three contributions in seconds a day, measuring stiffness at 20.000000 times the net expansion, finding the stiffness coefficient that removes the error, and placing the separator on dimensions versus material stiffness.

1. Notation and the Question

Setting: a balance wheel with moment of inertia I oscillates on a hairspring with torsional stiffness κ . I scales with the square of the wheel radius r , and κ is set by the elastic modulus E and the spring’s width b , thickness h and length L .

$$T = 2\pi\sqrt{\frac{I}{\kappa}}, \quad I \propto r^2, \quad \kappa = \frac{E b h^3}{12L}, \quad \frac{\Delta T}{T} = \frac{1}{2}(2\alpha_{\text{w}} - \alpha_E - 3\alpha_{\text{s}})$$

α_{w} and α_{s} are the expansion coefficients of wheel and spring, and α_E the temperature coefficient of stiffness (elastic modulus).The daily error is $-86400 \times \Delta T/T$ seconds (negative means losing).

Question: what is the main cause of a clock drifting with temperature?

2. Three Contributions

Computation 2.1: for three combinations, the daily error per K and its breakdown were computed.

combination	daily error (s/K)	wheel expansion	stiffness change	spring expansion
steel spring, steel wheel	−9.849600	−1.036800	−10.368000	+1.555200
steel spring, brass wheel	−10.454400	−1.641600	−10.368000	+1.555200
alloy spring, brass wheel	0.000000	−1.641600	+0.604800	+1.036800

Steel was given expansion 1.2×10^{-5} and stiffness coefficient -2.4×10^{-4} , the brass wheel expansion 1.9×10^{-5} , and the alloy spring expansion 0.8×10^{-5} and stiffness coefficient $+1.4 \times 10^{-5}$ (all per K, model values).

Observation 2.2

this is the core. The main actor is the change in stiffness. In all steel, the wheel's expansion lengthens the period, while the spring's expansion stiffens it and shortens the period, so the two cancel and leave a net 0.518400 seconds: the stiffness contribution, -10.368000 seconds, is 20.000000 times that and makes up almost all of the -9.849600 -second daily error.

Observation 2.3

the formula matches recomputed periods. Changing dimensions and stiffness by 0.001 K and recomputing the period gave slopes of -9.849602 , -10.454402 and -0.000000 seconds: within 1.9×10^{-6} seconds of the formula. Changing by a whole 1 K instead, the difference from the first-order formula was 1.809×10^{-3} seconds for all steel, the size of the second-order terms.

3. Removing the Error

Computation 3.1: for a brass wheel and a spring expanding at 0.8×10^{-5} , the stiffness coefficient giving zero daily error was found.

Observation 3.2

choosing the stiffness coefficient removes the error. At $\alpha_E = 2\alpha_w - 3\alpha_s = +1.4 \times 10^{-5}$, the daily error was 0 within 10^{-9} : a material that softens when warmed, like steel, cannot cancel it; choosing one that stiffens slightly cancels exactly the net of the wheel and spring expansions.

4. The Separator

Observation 4.1

the separator is whether one asks about the dimensions of the wheel and spring or about the stiffness of the material. Changes in dimension act in opposite directions in wheel and spring and cancel: the change in stiffness has nothing to cancel it and becomes 10.368000 seconds a day of error directly. The net expansion is only a twentieth of the stiffness contribution.

Observation 4.2

placed beside Paper 856.856 showed that what throws a pendulum off is where it is pushed, more than how hard. There the escapement's push upset the period. Here temperature upsets it, and the main actor is stiffness, not dimensions. In both, not the conspicuous quantity but another one decides the error.

Observation 4.3

what is not said. Real clock or alloy values, magnetism, position error, oil, and which material to use are not treated. What is shown is that the main actor in temperature drift is the change in stiffness.

5. Census and Closing

1. $\Delta T/T = \frac{1}{2}(2\alpha_w - \alpha_E - 3\alpha_s)$ (Section 1): **three contributions.**
2. all steel: **−9.849600 s a day per K** (Computation 2.1): **loses.**
3. stiffness contribution **−10.368000 s** (Computation 2.1): **nearly all of it.**
4. net expansion **0.518400 s** (Observation 2.2): **they cancel.**
5. stiffness **20.000000 times the net** (Observation 2.2): **the core.**
6. brass wheel: **−10.454400 s** (Computation 2.1): **barely changed.**
7. zero at $\alpha_E = +1.4 \times 10^{-5}$ (Observation 3.2): **the error vanishes.**
8. recomputed slopes match (Observation 2.3): **within 1.9×10^{-6} s.**
9. a whole 1 K shows second-order terms (Observation 2.3): **a caution on measuring.**
10. dimensions, or material stiffness (Observation 4.1): **the separator; Paper 856.**

Closing: What makes a hairspring clock drift with temperature is not expansion but the change in stiffness. Of the -9.849600 seconds a day per K in an all-steel model, the change in stiffness accounts for -10.368000 seconds, while the wheel and spring expansions cancel to a net 0.518400 seconds, because changes in dimension have an opposite partner, but the change in stiffness has nothing to cancel it. The stiffness contribution is 20.000000 times the net expansion, and an alloy with stiffness coefficient $+1.4 \times 10^{-5}$ paired with a brass wheel brings the error to 0: the key to removing drift lies not in dimensions but in the temperature change of the material's stiffness. Placed among the earlier papers: beside Paper 856's "where it is pushed, more than how hard", we placed that the main actor in temperature drift is stiffness, not dimensions. To be honest: the coefficients are model values, and magnetism, position error and oil are not included.

References

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