

Extractable Energy from a Rotating Black Hole Starts to Grow as the Square of the Spin and Reaches at Most 0.292893 of the Mass

— the extractable share is **0.001254** at $a = 0.1$, **0.034074** at 0.5 , **0.152684** at 0.9 and **0.292893** at the fastest, $a = 1$ — the separator is whether one asks about the speed of rotation or the area of the horizon — [Paper 887]

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— *From thinking to syncing.* —

Abstract

It is easy to assume that a rotating black hole yields energy in proportion to how fast it spins. This paper counts how the upper limit on extractable energy grows with spin, in two ways: from the irreducible-mass formula and from the horizon area. No new theorem or law is claimed. First, the extractable limit is the mass minus the irreducible mass: the irreducible mass is $\sqrt{(1 + \sqrt{1 - a^2})/2}$ times the mass: **0.965926** at $a = 0.5$ and **0.707107** at $a = 1$ (Section 2). Second, and this is the core. Even at the fastest spin, at most **0.292893** of the mass can be drawn: at $a = 1$ the extractable share is **0.292893**, matching $1 - 1/\sqrt{2}$ within 10^{-12} (Section 2). Third, for slow rotation, it starts to grow as the square of the spin: at $a = 0.1$ the share is **0.001254**, a ratio of **1.003142** to $a^2/8$; the ratio rises with spin, reaching **2.343146** at $a = 1$ (Section 3). Fourth, at half the spin, far less: at $a = 0.5$ the share is **0.034074**, only **1/8.595754** of the $a = 1$ amount (Section 3). Fifth, the irreducible mass is set by the horizon area: the irreducible mass recomputed from the horizon area as $\sqrt{A/16\pi}$ agreed with the formula within 10^{-12} for all eight spins (Section 2). Sixth, the separator is whether one asks about the speed of rotation or the area of the horizon. What sets the extractable limit is the horizon area, which does not decrease (Section 4). **Keywords:** Penrose process, Kerr black hole, irreducible mass, horizon area, rotational energy.

Scope of this paper (scope note): No new theorem or law is claimed: the Penrose process for extracting energy from a rotating black hole, the irreducible-mass formula, and that the horizon area does not decrease in classical processes, are all known in general relativity. Units are set: the gravitational constant and the speed of light are 1, and the spin parameter a is angular momentum divided by the square of the mass. No observations are claimed: how fast real objects spin, or whether energy is actually extracted, is not treated. Nothing beyond general relativity is entered: quantum effects and corrections from other theories are not included. Single-split efficiency is not entered: the limit on efficiency when one particle splits in two is not treated; what is counted is the limit on total extractable energy. Relation to earlier papers: Paper 73 anchored the “1/4” in black-hole entropy via the 4π solid angle, the Euler characteristic and equipartition. This paper stands beside it. There the horizon area was read as entropy. Here we count that the same area not decreasing sets the ceiling on extractable energy. What is added is laying out the extractable share for eight spins, checking that the fastest spin gives 0.292893, measuring the ratio to $a^2/8$ for slow spin and the ratio 8.595754 to half the spin, and placing the separator on rotation speed versus horizon area.

1. Notation and the Question

Setting: consider a black hole of mass M and spin parameter a ($0 \leq a \leq 1$), with the gravitational constant and speed of light set to 1. Let the irreducible mass M_{irr} be the mass that remains after all rotational energy is extracted, and let r_+ and A be the horizon radius and area.

$$\frac{M_{\text{irr}}}{M} = \sqrt{\frac{1 + \sqrt{1 - a^2}}{2}}, \quad r_+ = M(1 + \sqrt{1 - a^2}), \quad A = 8\pi M r_+, \quad M_{\text{irr}} = \sqrt{\frac{A}{16\pi}}$$

The extractable share is $1 - M_{\text{irr}}/M$. The irreducible mass was computed two ways, from the spin formula and from the horizon area.

Question: how much energy can be drawn from a rotating black hole?

2. The Extractable Share

Computation 2.1: for varying spin, the irreducible mass and extractable share were computed.

a	M_{irr}/M	extractable share	ratio to $a^2/8$
0.1	0.998746	0.001254	1.003142
0.3	0.988418	0.011582	1.029535
0.5	0.965926	0.034074	1.090374
0.7	0.925782	0.074218	1.211730
0.9	0.847316	0.152684	1.507987
0.99	0.755337	0.244663	1.997047
0.998	0.729114	0.270886	2.175784
1	0.707107	0.292893	2.343146

Observation 2.2

this is the core. Even at the fastest spin, at most 0.292893 of the mass can be drawn. At $a = 1$ the irreducible mass is 0.707107, and the extractable share 0.292893 matched $1 - 1/\sqrt{2}$ within 10^{-12} : the remaining 0.707107 cannot be drawn from rotation by any process.

Observation 2.3

the irreducible mass is set by the horizon area. Recomputed from the area as $\sqrt{A/16\pi}$, it agreed with the formula within 10^{-12} for all eight spins: the irreducible mass goes as the square root of the area, so if the area does not decrease, neither does the irreducible mass.

3. Spin and Extractable Energy

Computation 3.1: the extractable share was compared with $a^2/8$, and half spin with the fastest spin.

Observation 3.2

for slow rotation, it starts to grow as the square of the spin. At $a = 0.1$ the ratio of the extractable share to $a^2/8$ is 1.003142: as spin increases the ratio rises to 1.090374, 1.507987 and 2.343146, growing faster than the square.

Observation 3.3

at half the spin, far less. At $a = 0.5$ the extractable share is 0.034074, $1/8.595754$ of the 0.292893 at $a = 1$: halving the spin cuts the extractable amount not in half but to $1/8.595754$.

4. The Separator

Observation 4.1

the separator is whether one asks about the speed of rotation or the area of the horizon. Asking about rotation speed, it looks as if energy comes out in proportion to spin: asking about the horizon area, the area does not decrease during extraction, so the mass cannot fall below the irreducible mass. The area is what sets the limit.

Observation 4.2

placed beside Paper 73.73 anchored the “ $1/4$ ” in black-hole entropy to the horizon area. There the area was read as entropy. Here the same area not decreasing is counted as the ceiling on extractable energy.

Observation 4.3

what is not said. Spins and observations of real objects, quantum effects, and single-split efficiency are not treated. What is shown is that the extractable share starts as the square of the spin and reaches at most 0.292893.

5. Census and Closing

1. irreducible-mass formula (Section 1): **mass left after rotation is used up.**
2. irreducible mass **0.707107** at $a = 1$ (Computation 2.1): $1/\sqrt{2}$.
3. extractable limit **0.292893** (Observation 2.2): **the core.**
4. recomputed from area, agreeing (Observation 2.3): **within 10^{-12} .**
5. **0.001254** at $a = 0.1$ (Computation 2.1): **almost nothing.**
6. ratio to $a^2/8$: **1.003142** (Observation 3.2): **starts as the square.**
7. ratio **2.343146** at $a = 1$ (Computation 2.1): **faster than the square.**
8. **0.034074** at $a = 0.5$ (Computation 2.1): **half the spin.**
9. **1/8.595754** (Observation 3.3): **against the fastest spin.**
10. rotation speed, or horizon area (Observation 4.1): **the separator; Paper 73.**

Closing: The energy extractable from a rotating black hole starts to grow as the square of the spin and reaches at most 0.292893 of the mass. The irreducible mass is $\sqrt{(1 + \sqrt{1 - a^2})/2}$ times the mass; at $a = 1$, 0.707107 remains and 0.292893 can be drawn, because the irreducible mass goes as the square root of the horizon area, and the area does not decrease during extraction. For slow spin the share is close to $a^2/8$ (ratio 1.003142 at $a = 0.1$), and at half the spin, $a = 0.5$, only 1/8.595754 of the $a = 1$ amount can be drawn: energy does not come out in proportion to spin, but only beneath a ceiling set by the area. Placed among the earlier papers: beside the horizon area anchored in Paper 73, we placed that the area not decreasing is the ceiling on extractable energy. To be honest: all the formulas are known in general relativity, and real objects and quantum effects are not treated.

References

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