

A Resonating Body's Pitch Is Set Not by Length, as in a Pipe, but by Volume and Opening, Whatever the Shape

— doubling the volume multiplies the pitch by **0.707107**, doubling the opening radius by **1.426423** — the separator is whether one asks about the longest dimension or about the volume and the area of the opening —

[Paper 888]

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— *From thinking to syncing.* —

Abstract

It is easy to assume that an instrument body sounds low because it is large, or long. This paper counts that the pitch of a cavity with an opening is set by something other than the length that sets a pipe's resonance. No new theorem or law is claimed. First, a cavity with an opening sounds low while staying much smaller than the wavelength: a volume of 0.01 m^3 and an opening of radius 0.05 m give **163.086441 Hz**, with a wavelength of **2.103179 m** (Section 2). Second, and this is the core. With the same volume, different shapes sound the same pitch: a cube and a long box both give **163.086441 Hz**, while the half-wave pitch of the longest side is **796.032485 Hz** and **343.000000 Hz**, **2.320794** times apart (Section 2). Third, a larger volume lowers it and a larger opening raises it: doubling the volume gives **0.707107** times, doubling the opening radius **1.426423** times, and halving it **0.695353** times (Section 3). Fourth, the opening does not act through area alone: doubling the radius quadruples the area, but the neck length grows too, so the pitch rises only to **1.426423** times, just above $\sqrt{2}$ (Section 3). Fifth, the formula matches the moved system: moving the neck air as a mass on the cavity air as a spring and counting zero crossings gave **163.086529 Hz**, within **0.000088 Hz** of the formula (Section 2). Sixth, the separator is whether one asks about the longest dimension or about the volume and the area of the opening: a pipe resonance watches length; a cavity resonance watches volume and opening (Section 4). **Keywords:** Helmholtz resonance, resonating body, cavity, opening area, pipe resonance.

Scope of this paper (scope note): No new theorem or law is claimed: the Helmholtz resonance formula, the end correction of the neck by the opening radius, and that pipe resonances are set by length, are all known in acoustics. No real instruments are claimed: volume, opening and plate thickness are model values matching no guitar and no drum. Plate vibration is not included: coupling of the body's plates to the cavity and absorption at the walls are not included. The range is limited: only cavities much smaller than the wavelength are treated; the end-correction factor 1.7 is one representative choice. Relation to earlier papers: Paper 857 showed that pitch sees only the ratio of tension to weight, but the instrument bears the tension itself. This paper stands beside it. There, for a string, the ratio of tension to weight set the pitch. Here, for a cavity, the ratio of volume to opening sets the pitch, and shape does not matter. What is added is setting the cavity pitch beside the longest-side pitch for two shapes of equal volume, measuring pitch factors when volume and opening change, matching the formula against the moved neck and cavity, and placing the separator on longest dimension versus volume and opening.

1. Notation and the Question

Setting: a cavity of volume V has an opening of radius r in a plate of thickness t . The air in the opening (the neck) acts as a mass and the cavity air as a spring, with the neck length corrected by the opening radius.

$$f = \frac{c}{2\pi} \sqrt{\frac{A}{V L_{\text{eff}}}}, \quad A = \pi r^2, \quad L_{\text{eff}} = t + 1.7 r, \quad f_{\text{pipe}} = \frac{c}{2L_{\text{max}}}$$

We took $c = 343$ m/s and air density 1.2 kg/m^3 . f_{pipe} is the pitch set by the half wave of the longest side L_{max} . The cavity pitch was obtained from the formula and by moving the neck air as a mass on the cavity air as a spring in steps of 10^{-6} seconds and counting zero crossings.

Question: what sets the pitch at which a resonating body sounds?

2. Cavity Pitch and Length Pitch

Computation 2.1: for a cube and a long box of volume 0.01 m^3 , the cavity pitch and the longest-side pitch were computed (opening radius 0.05 m, plate 0.003 m).

shape	dimensions (m)	cavity pitch (Hz)	longest-side half wave (Hz)
cube	0.2154 each side	163.086441	796.032485
long box	$0.5 \times 0.2 \times 0.1$	163.086441	343.000000

Observation 2.2

this is the core. With the same volume, different shapes give the same cavity pitch. The cube and the long box both gave 163.086441 Hz, while the pitch set by the longest side is 796.032485 Hz and 343.000000 Hz, 2.320794 times apart. The cavity resonance does not watch length; it watches only volume and opening.

Observation 2.3

the cavity pitch is low while the cavity stays much smaller than the wavelength. The wavelength at 163.086441 Hz is 2.103179 m, far longer than the cavity: moving the neck and cavity and counting zero crossings gave 163.086529 Hz, 0.000088 Hz from the formula.

3. Volume and Opening

Computation 3.1: volume and opening radius were changed and the pitch factors computed.

change	pitch (Hz)	factor on original
volume $\times 2$	115.319529	0.707107
opening radius $\times 2$	232.630223	1.426423
opening radius $\times 0.5$	113.402727	0.695353

Observation 3.2

a larger volume lowers it and a larger opening raises it. Doubling the volume gives exactly $1/\sqrt{2}$ times, and doubling the opening radius 1.426423 times: doubling the radius quadruples the area, but the neck length corrected by the radius grows as well, so the pitch rises not to 2 times but just above $\sqrt{2}$.

4. The Separator

Observation 4.1

the separator is whether one asks about the longest dimension or about the volume and the area of the opening. A pipe resonance is set by length, and changing shape changes its pitch: a cavity resonance compresses the whole air as one spring, so it is set by volume and opening, and changing shape leaves its pitch alone. A body sounds low not because it is long, but because its opening is small for its volume.

Observation 4.2

placed beside Paper 857.857 showed that pitch sees only the ratio of tension to weight: for a string, that ratio set the pitch. Here, for a cavity, the ratio of opening area to volume and neck length sets it. In both, a ratio, not a conspicuous size, sets the pitch.

Observation 4.3

what is not said. Real instrument values, coupling to plate vibration, and wall absorption are not treated. What is shown is that a cavity's pitch is set by volume and opening, whatever the shape.

5. Census and Closing

1. $f = (c/2\pi)\sqrt{A/(VL_{\text{eff}})}$ (Section 1): **Helmholtz resonance.**
2. **163.086441** Hz (Computation 2.1): **volume 0.01 m³.**
3. **wavelength 2.103179** m (Observation 2.3): **far longer than the cavity.**
4. **same pitch for cube and long box** (Observation 2.2): **the core.**
5. **longest-side pitch 2.320794** times apart (Observation 2.2): **changes with shape.**
6. **volume $\times 2$: 0.707107** (Computation 3.1): **$1/\sqrt{2}$.**
7. **opening radius $\times 2$: 1.426423** (Computation 3.1): **the neck grows too.**
8. **opening radius $\times 0.5$: 0.695353** (Computation 3.1): **lower.**
9. **within 0.000088** Hz of the moved system (Observation 2.3): **independent.**
10. **longest dimension, or volume and opening** (Observation 4.1): **the separator; Paper 857.**

Closing: The pitch at which a resonating body sounds is set not by length, as in a pipe, but by the ratio of volume to opening, whatever the shape. A cavity of 0.01 m^3 with an opening of radius 0.05 m sounds at 163.086441 Hz whether cube or long box, and while the longest-side pitch differs 2.320794 times, the cavity pitch does not change, because the cavity air acts as one spring compressed as a whole, working through volume rather than length. Doubling the volume gives 0.707107 times and doubling the opening radius 1.426423 times, and the moved neck and cavity agree with the formula within 0.000088 Hz : a body sounds low not because it is long, but because its opening is small for its volume. Placed among the earlier papers: beside Paper 857's "pitch sees only the ratio of tension to weight", we placed that a cavity's pitch sees only the ratio of volume to opening. To be honest: dimensions are model values, plate vibration and absorption are not included, and only cavities smaller than the wavelength are treated.

References

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