

The Error in a Standing Tree's Volume Stacks Three Assumptions, and the Hardest to Measure, the Form Factor, Counts Most

— although diameter enters squared, the variance shares are **0.152381** for diameter, **0.238095** for height and **0.609524** for form factor, the form factor taking six tenths — the separator is whether one improves the precision of measurable quantities or of assumptions — [Paper 891]

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Abstract

It is easy to assume that measuring the diameter well makes a standing tree's volume accurate. This paper counts, by formula and by random trees, how the errors of three assumptions stack in a volume estimated without felling. No new theorem or law is claimed. First, the three errors stack as the square root of the sum of squares: with **0.02** for diameter, **0.05** for height and **0.08** for form factor, the relative error in volume is **0.102470** (Section 2). Second, the formula matches random trees: making **400000** trees with errors on the three quantities gave a relative error of **0.102684**, within **0.002** of the formula (Section 2). Third, and this is the core. The form factor counts most: variance shares are **0.152381** for diameter, **0.238095** for height and **0.609524** for form factor (Section 3). Fourth, diameter enters squared but its share does not reach the form factor's: a diameter error of **0.02** acts as **0.04** on volume, half the form factor's **0.08** (Section 3). Fifth, halving the form-factor error helps most: halving each error multiplies the volume error by **0.941124** for diameter, **0.906327** for height and **0.736788** for form factor (Section 3). Sixth, the separator is whether one improves the precision of measurable quantities or of assumptions: diameter and height can be measured, but the form factor is an assumption that cannot be checked without felling, and six tenths of the error lies there (Section 4). **Keywords:** stem volume, breast-height diameter, form factor, error propagation, variance share.

Scope of this paper (scope note): No new theorem or law is claimed: estimating standing volume as the product of form factor, breast-height basal area and height, that independent relative errors combine as the square root of the sum of squares, and that a squared quantity doubles its error, are all known. No real values are claimed: the three errors, diameter 0.30 m, height 20 m and form factor 0.45 are model values matching no species and no instrument. Correlation is not included: the three errors are taken as independent. Stands are not entered: only one tree's volume is treated; sampling a whole stand is not. Relation to earlier papers: Paper 581 showed that the age at which a forest takes up carbon fastest is not the age at which it holds the most. This paper stands beside it. There the amount stored in a forest was counted by stand age. Here we count the error in the volume of one tree, the source of that amount, when measured without felling. What is added is obtaining the stacked error by formula and by random trees, laying out variance shares and measuring the form factor's 0.609524, comparing the effect of halving each error, and placing the separator on measurable quantities versus assumptions.

1. Notation and the Question

Setting: a standing tree's volume V is estimated from form factor f , breast-height diameter D and height h . Each carries an independent relative error $\varepsilon_f, \varepsilon_D, \varepsilon_h$.

$$V = f \times \frac{\pi D^2}{4} \times h, \quad \frac{\sigma_V}{V} \approx \sqrt{(2\varepsilon_D)^2 + \varepsilon_h^2 + \varepsilon_f^2}$$

We set $\varepsilon_D = 0.02$, $\varepsilon_h = 0.05$, $\varepsilon_f = 0.08$, with true values $D = 0.30$ m, $h = 20$ m and $f = 0.45$ (volume 0.636173 m^3). The relative error was obtained from the formula and as the standard deviation over 400000 random trees with normal errors on the three quantities.

Question: where does the error in a volume estimated without felling come from?

2. How the Three Errors Stack

Computation 2.1: the relative error in volume was computed from the three relative errors.

Observation 2.2

the three errors stack as the square root of the sum of squares. The formula gives a relative volume error of 0.102470, and 400000 random trees gave 0.102684: within 0.002. The volume error is larger than any one of the three.

3. Shares and the Effect of Halving

Computation 3.1: variance shares, and the relative volume error when each error is halved, were computed.

quantity	error on volume	variance share	factor when halved
breast-height diameter	2×0.02	0.152381	0.941124
height	0.05	0.238095	0.906327
form factor	0.08	0.609524	0.736788

Observation 3.2

this is the core. The form factor counts most. The form factor’s variance share is 0.609524, six tenths: diameter enters squared, so its 0.02 error becomes 0.04 on volume, yet that is still half the form factor’s 0.08 , leaving its share at 0.152381.

Observation 3.3

halving the form-factor error helps most. Halving the form-factor error brings the volume error to 0.075498, 0.736788 times the original: halving the diameter error gives only 0.941124 times, and height 0.906327 times.

4. The Separator

Observation 4.1

the separator is whether one improves the precision of measurable quantities or of assumptions. Diameter and height can be measured without felling: the form factor is an assumption about how much the trunk tapers from a cylinder, which cannot be checked without felling. Six tenths of the error lies in that hard-to-measure assumption.

Observation 4.2

placed beside Paper 581.581 showed that the age of fastest carbon uptake is not the age of greatest storage. There the amount stored in a forest was counted by stand age. Here we counted where the error comes from when the volume of one tree, the source of that amount, is estimated without felling.

Observation 4.3

what is not said. Real species or instrument errors, correlated errors and whole-stand estimation are not treated. What is shown is that six tenths of the volume error comes from the form factor.

5. Census and Closing

1. $V = f \times \pi D^2 / 4 \times h$ (Section 1): a product of three assumptions.
2. relative error **0.102470** (Observation 2.2): formula.
3. **0.102684** at random (Observation 2.2): 400000 trees.
4. diameter share **0.152381** (Computation 3.1): small despite squaring.
5. height share **0.238095** (Computation 3.1): middling.
6. form-factor share **0.609524** (Computation 3.1): the core.
7. halving form factor: **0.736788** (Observation 3.3): helps most.
8. halving diameter: **0.941124** (Computation 3.1): helps little.
9. halving height: **0.906327** (Computation 3.1): helps somewhat.
10. measurable quantities, or assumptions (Observation 4.1): the separator; Paper 581.

Closing: The error in a standing tree's volume stacks three assumptions, and the hardest to measure, the form factor, counts most. Relative errors of 0.02, 0.05 and 0.08 in diameter, height and form factor give a volume error of 0.102470 (0.102684 at random), with the form factor taking a variance share of 0.609524, because diameter, even squared, acts only as 0.04, half the form factor's 0.08. Halving each error multiplies the volume error by 0.736788 for form factor, 0.906327 for height and 0.941124 for diameter: six tenths of the error lies not in what can be measured but in an assumption that cannot be checked without felling. Placed among the earlier papers: beside the forest storage counted in Paper 581, we placed where the error comes from in estimating one tree's volume without felling. To be honest: the three errors are model values taken as independent, and whole-stand estimation is not entered.

References

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