

Water's Price per Cubic Metre Traces a U with Usage, Cheapest at a Block Boundary That the Basic Charge Chooses

— a basic charge of **400** yen moves the cheapest to **10 m³**, **2500** yen moves it to **30 m³**, and at **4000** yen the price keeps falling up to **60 m³** — the separator is whether one asks about the total charge or divides by the cubic metres — [Paper 897]

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— *From thinking to syncing.* —

Abstract

Since block prices for water rise with usage, it is easy to assume that using less always makes each cubic metre cheaper. This paper counts, in a model tariff made of a basic charge and block prices that rise step by step, the price per cubic metre at each usage, and looks at where the cheapest usage falls. No new theorem or law is claimed. First, using little makes each cubic metre more expensive, not less: at 5 m³ it is **300.000000** yen, **1.714286** times the cheapest usage (Section 2). Second, using a lot also makes it more expensive: **200.000000** yen at 40 m³ and **216.666667** yen at 60 m³, **1.238095** times the cheapest (Section 2). Third, and this is the core. The cheapest usage falls on a block boundary: searching in steps of 0.01 m³, the cheapest is **175.000000** yen at 20 m³, above the 150 yen of the left block and below the 200 yen of the right (Section 2). Fourth, the basic charge decides which boundary: with a basic charge of 400 yen the cheapest is **140.000000** yen at 10 m³, with 2500 yen it is **233.333333** yen at 30 m³, and with 4000 yen the price keeps falling up to 60 m³, reaching **266.666667** yen (Section 3). Fifth, the U appears only with both a basic charge and rising blocks: with no basic charge the price per cubic metre only rose, and with a single block (150 yen) it only fell (Section 3). Sixth, the separator is whether one asks about the total charge or divides by the cubic metres: the total only grows with use, but dividing sets the thinning of the basic charge against the rising blocks, and their balance falls on a block boundary (Section 4). **Keywords:** water tariff, basic charge, block price, price per cubic metre, block boundary.

Scope of this paper (scope note): No new theorem or law is claimed: that water charges are set by a basic charge plus block prices by usage, that there are tariffs whose block prices rise with usage, and that an average price falls while the block price is below it and rises when above are all known in pricing and cost calculations. No real tariff is claimed: a basic charge of 1000 yen and block prices of 100, 150, 200 and 250 yen every 10 m³ are model values matching no water tariff. Other parts of the bill are not entered: sewer charges, basic charges by pipe size, tax, and billing periods are not included. Tariffs are not judged, which tariff is fair and how it affects saving water are not discussed. Relation to earlier papers: Paper 486 showed that the “exactly half” appearing in three fields is the vertex of a parabola, not physics. This paper stands beside it. There the best value came at the vertex of a smooth curve. Here the cheapest usage comes not at a smooth vertex but at a corner where the block price jumps. What is added is laying out the price per cubic metre at eight usages, measuring in steps of 0.01 m³ that the cheapest usage falls on a block boundary, measuring that changing the basic charge moves that boundary, and placing the separator on total versus per cubic metre.

1. Notation and the Question

Setting: the monthly charge $C(u)$ is the basic charge $B = 1000$ yen plus the usage u m³ times the block prices. Block prices are 100 yen for 0 to 10 m³, 150 yen for 10 to 20 m³, 200 yen for 20 to 30 m³, and 250 yen beyond 30 m³. The price per cubic metre is $C(u)/u$.

$$p(u) = \frac{C(u)}{u}, \quad \frac{dp}{du} = \frac{1}{u} (C'(u) - p(u))$$

$C'(u)$ is the block price at that usage. The average price p falls while the block price is below p and rises when above. Usage was moved from 1 m³ to 60 m³ in steps of 0.01 m³ to find the cheapest usage.

Question: does using less always make each cubic metre cheaper?

2. Usage and Price per Cubic Metre

Computation 2.1: the charge and the price per cubic metre were computed at eight usages.

usage (m ³)	charge (yen)	per cubic metre (yen)
5	1500	300.000000
10	2000	200.000000
15	2750	183.333333
20	3500	175.000000
25	4500	180.000000
30	5500	183.333333
40	8000	200.000000
60	13000	216.666667

Observation 2.2

this is the core. The price per cubic metre traces a U, and the cheapest usage falls on a block boundary. Searching in steps of 0.01 m³, the cheapest usage is 20 m³ at 175.000000 yen: that value is above the left block price of 150 yen and below the right block price of 200 yen. Before the boundary the block price is below the average, so the average falls; past it the block price is above the average, so it rises. The price at 5 m³ is 1.714286 times the cheapest, and at 60 m³ 1.238095 times.

3. Basic Charge and the Cheapest Usage

Computation 3.1: keeping block prices, the basic charge was changed and the cheapest usage searched for.

basic charge (yen)	cheapest usage (m ³)	per cubic metre (yen)
400	10	140.000000
1000	20	175.000000
2500	30	233.333333
4000	still falling at the searched 60	266.666667

Observation 3.2

the basic charge decides which boundary. With basic charges of 400, 1000 and 2500 yen, the cheapest usage moved to the block boundaries at 10, 20 and 30 m³, and its price always lay above the left block price and below the right: with a basic charge of 4000 yen the price was still falling at the searched 60 m³, where it is 266.666667 yen, above the top block price of 250 yen. As controls, with no basic charge the price only rose, and with a single block (150 yen) it only fell; neither made a U.

4. The Separator

Observation 4.1

the separator is whether one asks about the total charge or divides by the cubic metres. Asking about the total, the charge only grows with use: dividing by the cubic metres sets the thinning of the basic charge over usage against the rising block prices. Their balance falls on the block boundary, where the block price jumps across the average.

Observation 4.2

placed beside Paper 486.486 showed that the “exactly half” of three fields is the vertex of a parabola. There the best value came at the vertex of a smooth curve. Here the price curve has no vertex, and the cheapest usage comes at a corner where the block price jumps.

Observation 4.3

what is not said. Real water tariffs, sewer charges, basic charges by pipe size, tax, which tariff is fair, and effects on saving water are not treated. What is shown is that, in this model, the price per cubic metre traces a U, the cheapest usage falls on a block boundary, and the basic charge decides which.

5. Census and Closing

1. $p(u) = C(u)/u$ (Section 1): per cubic metre.
2. 300.000000 yen at 5 m³ (Computation 2.1): using little is dear.

3. **175.000000** yen at **20 m³** (Observation 2.2): **the core; a block boundary.**
4. **between 150 and 200** yen (Observation 2.2): **the average straddles the block prices.**
5. **216.666667** yen at **60 m³** (Computation 2.1): **using a lot is dear too.**
6. **1.714286** and **1.238095** times (Observation 2.2): **the two ends of the U.**
7. **10 m³** with a **basic charge of 400** yen (Computation 3.1): **140.000000** yen.
8. **30 m³** with a **basic charge of 2500** yen (Computation 3.1): **233.333333** yen.
9. **no basic charge, and a single block** (Observation 3.2): **no U.**
10. **total, or per cubic metre** (Observation 4.1): **the separator; Paper 486.**

Closing: The price of water per cubic metre traces a U with usage; the cheapest usage falls on a block boundary, and the basic charge decides which. With a basic charge of 1000 yen and rising block prices, the price was 300.000000 yen at 5 m³ and 216.666667 yen at 60 m³, and the cheapest was 175.000000 yen at the block boundary of 20 m³. There the average price lies between the left block's 150 yen and the right block's 200 yen. A basic charge of 400 yen moved it to 10 m³ and 2500 yen to 30 m³, and removing the basic charge or using a single block made the U disappear: the U comes from the thinning basic charge meeting the rising blocks. Placed among the earlier papers: beside Paper 486's "the best value comes at a smooth vertex", we placed a tariff whose cheapest usage comes at the corner of a block. To be honest: the basic charge and block prices are model values, and sewer charges and tax are not included.

References

1. Yamagishi, Y. (2026). *The "Exactly Half" in Three Fields Is the Vertex of a Parabola, Not Physics* [Paper 486]. YOI: <https://www.yamagishiyuuki.com/yoi/486>; DOI: 10.5281/zenodo.22759952

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