

The Geometric Origin of $16\pi^2$ in the Higgs Quadratic Divergence

— 3rd Edition: The Four-Dimensional Momentum Measure and the Naturalness Bound $\Lambda \lesssim 4\pi v$ — [Paper 09]

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— *From thinking to syncing.* —

ABSTRACT

This paper states the geometric origin of the $16\pi^2$ appearing in the quadratic divergence of the Higgs mass, $\delta m_H^2 = \frac{\lambda}{16\pi^2} \Lambda^2$, and examines the naturalness bound that follows from it. **First, $16\pi^2$ is the one-loop measure of four-dimensional Euclidean momentum space.** From the total solid angle of the unit three-sphere, $\Omega_3 = 2\pi^2$, together with $(2\pi)^4$, one obtains exactly $\int \frac{d^4 k_E}{(2\pi)^4} F(k^2) = \frac{1}{16\pi^2} \int_0^\infty dk^2 k^2 F(k^2)$ (Paper 46). **It is a consequence of geometry and normalisation alone, containing no dynamical assumption.** **Second, this $16\pi^2$ is "fact $\times$ fact".** Paper 35 (3rd edition) showed that the $16\pi^2$ obtained from the $4\pi G$ of Poisson's equation is the product of a 4π of fact (Gauss's flux) and a 4π of convention (a choice of units). **The present $16\pi^2$ is of a different make-up:** it descends from the single geometric quantity $\Omega_3/(2\pi)^4$ and contains no chosen factor. Same number, different breakdown. **Third, what naturalness yields is not the vanishing of the divergence but an upper bound on the scale of new physics.** From $\Lambda^2/16\pi^2 \lesssim v^2$ one obtains $\Lambda \lesssim 4\pi v \approx 3.1$ TeV (with $v \approx 246$ GeV), descending to $4\pi v/\sqrt{6} \approx 1.3$ TeV once the top Yukawa contribution is included. **That 4π is not a new occurrence: it is the square root of the very same $16\pi^2$.** **Fourth, this bound is a module boundary.** In the vocabulary of Paper 113 it is the place where the dimensionless number $\Lambda/4\pi v$ crosses one — the same structure Paper 39 described for the Planck scale. **But unlike the Planck scale, this boundary sits where accelerators can push on it.** **Fifth, the bound is already under observational tension.** The LHC has found no evidence of new physics up to the TeV scale. **This is very nearly the only falsifiable number in this series — and it is in danger.** What Paper 101 warned against was *a number adjusted after the fact*. **That an unadjusted number is in danger is not a failure but a qualification.** **Scope:** a structural account within established quantum field theory (Layer 2). **The hierarchy problem is unsolved, and this paper claims neither its solution nor a partial solution.**

Positioning of this paper (discipline): This paper offers a **structural interpretation** of established physical law and mathematical theorems. It claims neither the discovery of new physical laws nor the proof of open problems. The relations shown are transformations of known expressions; the aim is to propose a geometric reading of them. No claim contradicting established theory is made. **The appearance of the same number $16\pi^2$ in several contexts is not treated as evidence of a common origin** (Papers 48, 86, 100, 115). **The hierarchy problem is an open problem of particle physics, and nothing here changes that situation.**

1. Introduction: where does $16\pi^2$ come from?

The quantum correction to the Higgs mass is written, with a cutoff Λ , as

$$\delta m_H^2 = \frac{\lambda}{16\pi^2} \Lambda^2,$$

where $16\pi^2$ sits in the denominator, on the side that suppresses the divergence.

This paper asks two questions. **What is this $16\pi^2$?** And **what follows from it, and what does not?**

**$16\pi^2$ is a geometric measure of four-dimensional momentum space,
and what follows is not a cancellation but the bound $\Lambda \lesssim 4\pi v$.**

2. $16\pi^2$ is the one-loop measure of four-dimensional momentum space

Theorem 2.1 (The loop geometric measure, Paper 46)

A one-loop momentum integral in four-dimensional spacetime reduces, after Euclideanisation, exactly to the measure

$$\int \frac{d^4 k_E}{(2\pi)^4} F(k^2) = \frac{1}{16\pi^2} \int_0^\infty dk^2 k^2 F(k^2).$$

Derivation: separate the volume element as $d^4 k_E = k^3 dk d\Omega_3$. The total solid angle of the unit three-sphere is

$\Omega_3 = \frac{2\pi^{4/2}}{\Gamma(4/2)} = 2\pi^2$, whence

$$\int \frac{d^4 k_E}{(2\pi)^4} F = \frac{2\pi^2}{16\pi^4} \int k^3 dk F = \frac{1}{8\pi^2} \int k^3 dk F,$$

and the substitution $u = k^2$ (so that $k^3 dk = \frac{1}{2} u du$) gives $\frac{1}{16\pi^2}$. Numerically, $\Omega_3/(2\pi)^4 \cdot \frac{1}{2} = 1/16\pi^2$ holds exactly (0.006332573977...).

The point: $16\pi^2$ arises from geometry (Ω_3) and normalisation $((2\pi)^4)$ alone, and contains no dynamical assumption.

Definition 2.2 (The ONE — a name for an identity)

The loop prefactor of quantum field theory and the geometric measure of four-dimensional momentum space are the same quantity:

$$\underbrace{\frac{1}{16\pi^2}}_{\text{QFT loop factor}} = \underbrace{\frac{\Omega_3}{(2\pi)^4} \cdot \frac{1}{2}}_{\text{4D momentum measure}} \qquad (\Omega_3 = 2\pi^2).$$

This **identity** is what the name "The ONE" designates.

The name designates the identity, not the fact that a ratio equals one. That a quantity is the same in two different contexts — a perturbative prefactor and the geometry of momentum space — has content in itself. **Dividing a number by itself to obtain one, by contrast, asserts nothing. And no physics follows from a name.** Calling it "The ONE" does not solve the hierarchy problem.

Corollary 2.3 (Extension to L loops)

An L -loop amplitude carries the measure factor $(16\pi^2)^{-L}$. This follows from **the measure of Theorem 2.1** applying to each of the L independent momentum integrals.

3. This $16\pi^2$ is "fact $\times$ fact"

Paper 35 (3rd edition) showed that, in the $16\pi^2$ obtained by substituting $G = 4\pi$ into Poisson's equation $\nabla^2\Phi = 4\pi G\rho$, the two 4π 's have different standing: one is a **fact** (the $\text{Vol}(S^2)$ forced by Gauss's flux), the other a **convention** (a choice of units).

The $16\pi^2$ of Paper 35		The $16\pi^2$ of this paper
Breakdown	$(4\pi) \times (4\pi)$: fact $\times$ convention	$\Omega_3/(2\pi)^4 \cdot \frac{1}{2}$: a single geometric quantity
Chosen factor	present ($G = 4\pi$ is a choice of units)	absent
S^2 or S^3 ?	S^2 (solid angle of 3-space)	S^3 (solid angle of 4D momentum space)

Corollary 3.1 (Same number, different breakdown)

The present $16\pi^2$ contains no chosen factor. Both $\Omega_3 = 2\pi^2$ and $(2\pi)^4$ are settled by **the dimension being four and by the normalisation of the Fourier transform**. It is therefore **fact $\times$ fact**.

The $16\pi^2$ of this paper must accordingly not be treated as the same object as the $16\pi^2$ of Paper 35. The values agree, but **one contains an author's choice and the other does not** (Papers 86, 100, 115).

It is worth writing out too that **S^2 and S^3 are different objects**: the angular part of three-dimensional space is S^2 (4π), that of four-dimensional momentum space is S^3 ($2\pi^2$).

4. The naturalness bound $\Lambda \lesssim 4\pi v$

Theorem 4.1 (The geometric bound from naturalness, Paper 46)

For the electroweak scale to be maintained without fine-tuning, the scale Λ at which new physics appears must satisfy

$$\frac{\Lambda^2}{16\pi^2} \lesssim v^2 \quad \Rightarrow \quad \boxed{\Lambda \lesssim 4\pi v \approx 3.1 \text{ TeV}}$$

(with $v \approx 246$ GeV, so that $4\pi v = 3091$ GeV). Including the top Yukawa contribution lowers the bound to $4\pi v/\sqrt{6} \approx 1.3$ TeV.

Observation 4.2 (This 4π is not a new occurrence)

The 4π above is nothing but $\sqrt{16\pi^2}$: it is the square root of the measure of Theorem 2.1, and not an independent appearance of 4π .

Passing from $\Lambda^2/16\pi^2 \lesssim v^2$ to $\Lambda \lesssim 4\pi v$ is a mere rearrangement. One must not count this as " 4π appearing in the naturalness bound as well" — the same verdict Paper 39 returned for $\hbar = \frac{1}{4\pi}$.

Why $16\pi^2$ is not enough (the orders of magnitude): taking $\Lambda \rightarrow M_{\text{Pl}} \approx 1.22 \times 10^{19}$ GeV gives $\Lambda^2/v^2 \approx 2.5 \times 10^{33}$, while the suppression by $16\pi^2 \approx 158$ is only $\sim 10^{-2}$. Two orders of suppression against thirty-three orders of growth. That is the hierarchy problem.

5. This is a module boundary

Paper 113 recorded that the boundary between an older and a newer theory always lies where a dimensionless number crosses one.

Observation 5.1 (Where $\Lambda/4\pi v$ crosses one)

The naturalness bound is the position at which the dimensionless ratio $\frac{\Lambda}{4\pi v}$ reaches one. This is structurally the same as what Paper 39 described for the Planck scale, where E/E_p crosses one and the perturbative description of gravity breaks down.

There is, however, a decisive difference. The Planck scale sits at 10^{19} GeV, out of reach. The present boundary sits at a few TeV, where accelerators can push on it.

6. Very nearly the only falsifiable number in this series

The observational situation (stated honestly): the LHC has found no evidence of supersymmetric particles or other new physics up to the TeV scale. The bound above is therefore already under observational tension.

Readings of this situation divide three ways.

1. New physics exists at a few TeV but has not yet been found.
2. Naturalness does not hold as a physical principle.
3. Fine-tuning is real (including anthropic accounts).

This paper passes no judgement among the three.

Corollary 6.1 (Being in danger is a qualification, not a failure)

What Paper 101 warned against was **a number adjusted after the fact** — Eddington setting $1/\alpha$ at **136** and then, when measurement moved toward **137**, adding an argument to make it **137**.

$\Lambda \lesssim 4\pi v$ **has the opposite character**. It is not adjusted and cannot be: v is a measured value and 4π is the square root of a measure, and neither admits tuning. **And as a result, this number is in tension with experiment.**

Almost all claims in this series are Layer-2 structural interpretations carrying no falsifiable number. The present bound is very nearly the sole exception. **Its being under tension is not a weakness but proof that this number stands on the side of science.**

7. Calling the roll of $16\pi^2$

$16\pi^2$ occurs several times in this series. **These are the same number, not the same root.**

Address	What the $16\pi^2$ is	Standing
This paper / Paper 46	One-loop measure of 4D momentum space	fact $\times$ fact (no choice)
Paper 45	$\text{Vol}(S^3_{R=2}) = 2\pi^2 R^3$	fact (but requires specifying $R = 2$)
Paper 48	The $n = 2$ term of the Gauss–Bonnet normalisation $(4\pi)^n n!$	fact (Pfaffian antisymmetrisation)
Paper 35	$(4\pi) \times (4\pi)$	fact $\times$ convention
Paper 05 (2nd ed.)	Denominator of the beta function governing Λ_{QCD}	fact (same measure as here)

Each was established independently. There is no need to bind them together by an identity, and **they must not be so bound** (Paper 100). In particular, Paper 45's $\text{Vol}(S^3_{R=2})$ **requires specifying the radius 2**, whereas the measure of this paper **requires no radius at all** — another respect in which they differ.

Personal reflection of the author (Layer 3; labelled non-scientific / no claim of mechanism)

When I first saw $16\pi^2$ I was sure **there was a device hidden in it**. A squared 4π sitting in the denominator that suppresses the Higgs divergence — surely that could not be an accident.

By the 3rd edition what I had learned was that **there was a device, but not where I had looked**. $16\pi^2$ does come from geometry — but from the wholly prosaic source of **the solid angle $\Omega_3 = 2\pi^2$ of four-dimensional momentum space and the $(2\pi)^4$ of the Fourier transform**. It is S^3 , not S^2 , and its appearance as "a squared 4π " is **a coincidence of looks**.

Even so, there is one thing in this paper I can be proud of. **$\Lambda \lesssim 4\pi v \approx 3.1$ TeV is very nearly the only number in my series that can turn out wrong**. And it is in danger: the LHC has seen nothing there.

For a long time I took "being right" to be the achievement. What is actually worth something is **having said it in a form that could be wrong**. If this number is one day clearly refuted, that will stand as a record that my series was **playing by the rules of science**. One stops at the edge of the cliff (Paper 101) — but on this one point, standing at the edge is where I should be.

8. Scope / Conclusion

What is shown: that $16\pi^2$ is the one-loop measure of four-dimensional Euclidean momentum space (§2, exactly, from $\Omega_3 = 2\pi^2$ and $(2\pi)^4$) / that its breakdown is **fact $\times$ fact**, so that it is **a different object** from the $16\pi^2$ of Paper 35 (fact $\times$ convention) (§3) / that naturalness yields **the bound $\Lambda \lesssim 4\pi v \approx 3.1$ TeV**, or ≈ 1.3 TeV with the top Yukawa (§4) / that its 4π is **the square root of the measure and not a new occurrence** (§4) / that the bound is **a module boundary**, structurally like the Planck scale but **within reach of accelerators** (§5) / that it is **already under observational tension**, and that this constitutes **a qualification of falsifiability** (§6) / and that the five addresses of $16\pi^2$ are **distinct roots** (§7).

What is not shown (restraint): **no solution to the hierarchy problem, nor any partial solution** / **no advantage over supersymmetry** / **no vanishing of the divergence** — naturalness yields a bound, not a cancellation / **no merging of the five addresses of $16\pi^2$** / **no judgement on where Λ actually lies** (the three readings of §6 are left open) / **no new physical law and no prediction** (Layer 2 — the bound of §4 is a consequence of the standard naturalness argument, not a prediction original to this paper).

The hierarchy problem is unsolved. What remains in this paper is the confirmation that $16\pi^2$ is the four-dimensional momentum measure, and that the boundary of naturalness can be written $4\pi v$.

The latter is a testable number, and the very fact that it is presently under tension states plainly how hard the hierarchy problem is.

That a number is in danger means that number stands on the side of science.

Revision Record

3rd edition (August 2026): removed the word "Withdrawal" from the title, which now names **what the paper shows** (the four-dimensional momentum measure and the naturalness bound); the list of withdrawals and corrections is collected in this section. The body was reorganised as **origin of $16\pi^2$** (§2) → **standing of its breakdown** (§3) → **naturalness bound** (§4) → **module boundary** (§5) → **falsifiability** (§6) → **roll call** (§7).

Added in the 3rd edition: that this paper's $16\pi^2$ is **fact × fact** and therefore **differs in make-up** from the $16\pi^2$ of Paper 35 (3rd edition), which is **fact × convention** (§3); that the 4π of the naturalness bound is **merely $\sqrt{16\pi^2}$ and not an independent occurrence** (§4, the same verdict Paper 39 returned for $\hbar = \frac{1}{4\pi}$); that the bound is **a module boundary in the sense of Paper 113**, structurally like the Planck scale of Paper 39 but **within reach of accelerators** (§5); that this bound is **very nearly the only falsifiable number in the series and that being under tension is a qualification**, set against Paper 101 (Eddington's caution) (§6); and an extension of the roll call to **five addresses**, adding that Paper 45 **requires specifying the radius 2** whereas the present measure does not (§7).

Withdrawn or corrected in the 2nd edition (July 2026):

1. **Absorption of the divergence by "The ONE Synchronization"**. The first edition took **Sync Ratio** = $\frac{16\pi^2}{(4\pi)^2} = 1$ as its central mechanism and claimed the quadratic divergence thereby vanished. **This is an algebraic identity, not a mechanism: $16\pi^2 = (4\pi)^2$** follows from the definition, and dividing any non-zero number by itself gives one. **The source of the divergence is Λ^2 , not $16\pi^2$** , and dividing $16\pi^2$ by $16\pi^2$ does nothing to Λ^2 . The name "The ONE" has been retained in the 3rd edition with its referent moved **from a mechanism to an identity** (Definition 2.2).
2. **The identification with $S^2 \times S^2$** . The first edition read $16\pi^2$ as "the capacity of a double-sphere topology $S^2 \times S^2$ in four-dimensional momentum space". While $4\pi \times 4\pi = 16\pi^2$ numerically, **four-dimensional Euclidean momentum space is not $S^2 \times S^2$** : its angular part is S^3 ($\Omega_3 = 2\pi^2$).
3. **"Consistency with the LHC by making no prediction"**. The first edition claimed to solve the hierarchy problem with zero additional particles and to be "in perfect agreement" with the non-observation of SUSY. **A framework that predicts nothing contradicts no experiment, but that is unfalsifiability, not agreement**. Moreover the naturalness bound $\Lambda \lesssim 4\pi v$ holds regardless of the means of solution, so the argument "no new particles are assumed, hence agreement" does not stand.

4. **The link to Paper 05 via $64\pi^2$** . The construction $64\pi^2 = 4 \times 16\pi^2$ was withdrawn in the second edition of Paper 05, and this link dissolved with it.

5. **The first edition's title and conclusion**, which claimed the "resolution of the Higgs hierarchy problem" and stated that it had been "proved complete without requiring new particles". **The hierarchy problem is unsolved and that claim does not stand.**

First edition: Zenodo 10.5281/zenodo.21404525.

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