The Same c³ Counts Differently: Black-Body Radiation's 3 Counts Dimensions, the Planck Length's Does Not

— counted in d dimensions, the exponent of c in the black-body radiation density is d, whereas the exponent of c in the Planck length times $d-1$ is -3 for every d from 2 to 6 — the separator is whether one counts by the volume of k space or by the exponents of the reduction — [Paper 902]

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— From thinking to syncing. —

Abstract

Paper 116 showed that the 3 in the c^3 of the Planck length is not the number of space dimensions but the weight of time.This paper shows one place where the same c^3 does count space dimensions: the radiation density of a black body.No new theorem or law is claimed. First, the exponent of T in the radiation density is $d+1$——the exponents fitted by numerical integration for $d=1$ to 4 are 2.000000, 3.000000, 4.000000, 5.000000, and the 4 of Stefan and Boltzmann is $d+1$ at $d=3$ (Section 2). **Second, and this is the core. The exponent of c in the radiation density is d——in the same integral the exponent of c is 1.000000, 2.000000, $\mathbf{3.000000}$, 4.000000; the 3 of c^3 is there because k space has three dimensions** (Section 2). **Third, the 4π of k space lives in the coefficient——the coefficient for $d=3$ with 2 polarisations is $2\times S_2/(2\pi)^3\times\Gamma(4)\zeta(4)=\mathbf{0.657974}=\pi^2/15$, where $S_2=4\pi$ is the area of the sphere in k space. The SI defining constants give $\sigma=\mathbf{5.670374419\times10^{-8}}$ W/m²/K⁴** (Section 3). **Fourth, the $1/4$ of the flux is also a division by 4π——$F=cu\times$ (the projected hemisphere $\int\cos\theta d\Omega$ divided by the full sphere S_{d-1}), which is $\mathbf{0.250000}=\pi/4\pi$ at $d=3$, 0.318310 at $d=2$ and 0.212207 at $d=4$** (Section 3). **Fifth, the 3 of the Planck length is 3 in every dimension——solving the balance of units for $d=2$ to 6, the exponent of c times $d-1$ is $\mathbf{-3.000000}$ throughout, in contrast to the c^d of the radiation density** (Section 4). **Sixth, the separator is whether one counts by the volume of k space or by the exponents of the reduction——in the former the power of c counts dimensions; in the latter it counts the weight of time. The same 3 has two identities** (Section 5). **Keywords**: black-body radiation, radiation density, density of states, exponent of c, Planck length, space dimension.

Scope of this paper (scope note): No new theorem or law is claimed——Planck's radiation law, the Stefan--Boltzmann law, the d-dimensional density of states proportional to ω^{d-1}, and the radiation density proportional to $T^{d+1}/(\hbar c)^d$ are all known (Planck 1901, Boltzmann 1884).The number of polarisations is left out of the coefficient——it is a constant that does not enter the exponents of T and c, so it is omitted in the integrals that fit the exponents and inserted as 2 only when the $d = 3$ coefficient is checked.Real bodies are not treated——grey bodies, radiation out of equilibrium and real sources are not included.No world with other than 3 dimensions is claimed——the d in the formulas is varied to read the exponents, and no such world is said to exist.Relation to earlier papers: Paper 116 showed that the 3 in the c^3 of the Planck length is the weight of time (the T^{-1} of $\hbar$ and the T^{-2} of G) and that the space dimension sits in the denominator $1/n$——here one place where the same c^3 counts space dimensions is set alongside, and the two identities of the 3 are counted.Paper 265 showed that the 3 in the T^3 of low-temperature heat capacity is the dimension, coming from the density of states ω^{d-1}——that the exponent of T in the radiation density is $d+1$ comes from the same density of states. What is new here is the exponent of c, not of T.Paper 222 showed that the position of Wien's peak moves with the choice of variable and only the total σT^4 is one——here the 4π and the c^3 inside the coefficient of that total are read.Paper 114 counted the objects that select a dimension and those that do not——the exponent of c in the radiation density selects, and the exponent of c in the Planck length does not.What is added is fitting the exponents of T and c by numerical integration for $d = 1$ to 4, counting the 4π of k space and the $1/4$ of the flux inside the coefficient, solving the exponent of c in the Planck length for $d = 2$ to 6 as a contrast, and placing the separator on the volume of k space versus the exponents of the reduction.

1. Notation and the Question

Setting: the radiation density of light in a d-dimensional cavity is the product of the Planck distribution and the density of states obtained by rewriting the k-space volume element $d^dk/(2\pi)^d$ with $\omega = ck$.With $S_{d-1} = 2\pi^{d/2}/\Gamma(d/2)$ the area of the sphere in k space,

$$u_d(T) = \frac{S_{d-1}}{(2\pi)^d}\int_0^\infty \frac{\hbar\omega\,\omega^{d-1}}{c^d\left(e^{\hbar\omega/kT}-1\right)}\,d\omega = \frac{S_{d-1}\,\Gamma(d+1)\zeta(d+1)}{(2\pi)^d}\,\frac{(kT)^{d+1}}{(\hbar c)^d}$$

The closed form on the right was not used; the integral on the left was evaluated numerically with $\hbar = k = 1$, and the exponents were fitted from the logarithm of the ratio when T or c is doubled.As a contrast, the exponents making a length out of $[\hbar] = ML^2T^{-1}$, $[G_d] = M^{-1}L^dT^{-2}$ and $[c] = LT^{-1}$ were solved from the linear equations of the balance of units.

Question: what does the 3 in the c^3 of the black-body radiation density count?

2. Fitting the Exponents

Computation 2.1: for $d = 1$ to 4, the exponents of T and c and the value of the integral were obtained.

d	exponent of T	exponent of c	$\int_0^\infty x^d/(e^x-1)dx$ (numerical)	$\Gamma(d+1)\zeta(d+1)$
1	2.000000	1.000000	1.644934	1.644934
2	3.000000	2.000000	2.404114	2.404114
3	4.000000	3.000000	6.493939	6.493939
4	5.000000	4.000000	24.886266	24.886266

> **Observation 2.2**
>
> this is the core. The exponent of c in the radiation density is d.The exponent of T is $d+1$ and that of c is d, both coming from the k-space volume element $k^{d-1}dk = \omega^{d-1}d\omega/c^d$——the 4 of the Stefan--Boltzmann T^4 is $d+1$ at $d=3$, and the 3 of c^3 is there because k space has three dimensions. The integral agreed with $\Gamma(d+1)\zeta(d+1)$.

3. The 4π in the Coefficient and the $1/4$ of the Flux

Computation 3.1: the coefficient for $d = 3$ with 2 polarisations, the Stefan--Boltzmann constant from the SI defining constants, and the flux factor were obtained.

quantity	value
coefficient $2 \times S_2/(2\pi)^3 \times \Gamma(4)\zeta(4)$	0.657974
$\pi^2/15$	0.657974
$S_2 = 4\pi$	12.566371
coefficient of u (SI, J/m³/K⁴)	7.565733×10^{-16}
coefficient of $\sigma = cu/4$ (W/m²/K⁴)	$5.670374419 \times 10^{-8}$
flux factor for $d = 2, 3, 4$	0.318310, 0.250000, 0.212207

Observation 3.2

the 4π of k space lives in the coefficient.$\pi^2/15$ is $2 \times 4\pi \times 6 \times (\pi^4/90)/(8\pi^3)$, and the 4π is not a solid angle but the area of the unit sphere in k space——this 4π is not chosen; it is the number that says k space has three dimensions. With the SI defining constants, $\sigma = 5.670374419 \times 10^{-8}$ W/m²/K⁴ comes out.

Observation 3.3

the $1/4$ of the flux is also a division by 4π.The flux through a surface is $F = cu \times \int_{\text{hemisphere}} \cos\theta d\Omega/S_{d-1}$, which at $d = 3$ is the projected hemisphere π divided by the full sphere 4π, 0.250000——at $d = 2$ it is $1/\pi = 0.318310$ and at $d = 4$ it is 0.212207; the $1/4$ too is a number that selects the dimension.

4. Contrast: The Exponent of c in the Planck Length

Computation 4.1: the exponents making $\hbar^a G_d^b c^e$ a length were solved for $d = 2$ to 6.

d	a	b	e	$e \times (d-1)$
2	1.000000	1.000000	−3.000000	−3.000000
3	0.500000	0.500000	−1.500000	−3.000000
4	0.333333	0.333333	−1.000000	−3.000000
5	0.250000	0.250000	−0.750000	−3.000000
6	0.200000	0.200000	−0.600000	−3.000000

Observation 4.2

the 3 of the Planck length is 3 in every dimension.The length is $(\hbar G_d/c^3)^{1/(d-1)}$, and the exponent of c inside the bracket stays 3 as d is varied——the c^d of the radiation density moves with d; the c^3 of the Planck length does not. In the same c^3, one 3 is the dimension of k space and the other is the sum of the time weights of $\hbar$ and G.

5. The Separator

Observation 5.1

the separator is whether one counts by the volume of k space or by the exponents of the reduction.Counting by the volume of k space, c is the conversion from ω to k, applied d times, so its power counts dimensions——counting by the exponents of the reduction, c is the conversion that removes the time weights of $\hbar$ and G, and their sum 3 does not depend on the dimension.

Observation 5.2

placed among the earlier papers.Paper 116 showed that the 3 in the c^3 of the Planck length is the weight of time——here a place where the same c^3 counts dimensions is set alongside. Paper 265 showed that the 3 in the T^3 of heat capacity is a dimension coming from the density of states; the T^{d+1} of the radiation density comes from the same density of states. Paper 222 showed that the total σT^4 is one; here the 4π and the c^3 inside its coefficient were read. Paper 114 counted objects that select a dimension and those that do not; the power of c in the radiation density selects, and the power of c in the Planck length does not.

Observation 5.3

what is not said.The number of polarisations, radiation from real bodies, grey bodies, non-equilibrium, and the existence of worlds with other than 3 dimensions are not treated——what is shown is that the power of c in the radiation density counts d, and the power of c in the Planck length does not.

6. Census and Closing

1. $u_d \propto T^{d+1}/(\hbar c)^d$ (Section 1): **from the k-space volume element.**
2. **exponents of T: $2, 3, 4, 5$** (Computation 2.1): **$d+1$.**
3. **exponents of c: $1, 2, 3.000000, 4$** (Computation 2.1): **the core.**
4. **agreement with $\Gamma(d+1)\zeta(d+1)$** (Computation 2.1): **6.493939 at $d=3$.**
5. **coefficient $0.657974 = \pi^2/15$** (Observation 3.2): **the 4π of k space.**
6. $\sigma = 5.670374419 \times 10^{-8}$ (Observation 3.2): **from the defining constants.**
7. **flux factor 0.250000** (Observation 3.3): **$\pi/4\pi$; 0.318310 at $d=2$.**

8. **Planck length $e \times (d-1) = -3.000000$** (Computation 4.1): **unchanged for $d = 2$ to 6.**
9. **two identities of the 3** (Observation 4.2): **the dimension of k space, and the weight of time.**
10. **volume of k space, or exponents of the reduction** (Observation 5.1): **the separator; Papers 116, 265, 222, 114.**

Closing: **The same c^3 counts differently: in the radiation density of a black body the 3 is the number of space dimensions, while in the Planck length it is 3 in every dimension.Integrating the radiation density numerically in d-dimensional k space, the exponent of T was $d+1$ and that of c was d; the $d = 3$ coefficient $\pi^2/15 = 0.657974$ contained the area 4π of the sphere in k space, and the $1/4$ of the flux was the projected hemisphere π divided by the full sphere 4π——c is the conversion from ω to k, applied once for each dimension of k space.In the contrasting Planck length, the exponent of c times $d-1$ was -3.000000 for every d from 2 to 6——there the 3 is the sum of the time weights of $\hbar$ and G, independent of the dimension.Placed among the earlier papers——beside Paper 116's "the 3 of c^3 is not a dimension", one place where it is a dimension was set. That the power of T counts the dimension comes from the same density of states as Paper 265.To be honest——the number of polarisations was left out of the coefficient, and no world with other than 3 dimensions is said to exist.**

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