The Angle a Foucault Pendulum Turns in a Day Is 2π Minus the Solid Angle of the Sky Above the Latitude Circle

— the angle $2\pi \sin\varphi$ through which the plane of swing turns in one sidereal day is 2π minus the change of direction under parallel transport around the latitude circle (the solid angle of the cap, $\Omega = 2\pi(1 - \sin\varphi)$), and does not depend on the pendulum's period — the separator is whether the day is counted in sidereal or in solar days — [Paper 903]

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— *From thinking to syncing.* —

Abstract

The angle $2\pi \sin\varphi$ that a Foucault pendulum turns in a day is usually taken as a number that comes out of the Coriolis force.This paper obtains the same angle in three ways: by numerical integration of the equations of motion, by parallel transport along the latitude circle, and from the solid angle of the cap above the latitude circle, and counts that they are one angle.No new theorem or law is claimed. First, stepping the equations of motion gives $2\pi \sin\varphi$——the turn in one sidereal day is 206.487516 degrees at latitude 35, 271.076195 at 48.85, 359.999994 at the pole and 0 at the equator, within 10^{-7} rad of $2\pi \sin\varphi$ (Section 2). **Second, the change of direction under parallel transport along the latitude circle is the solid angle of the cap——after one circuit the direction has changed by 2.679298 rad at 35 degrees and 1.552013 rad at 48.85, agreeing within 10^{-6} with the numerically integrated solid angle $2\pi(1 - \sin\varphi)$ of the cap above the circle** (Section 3). **Third, and this is the core. The three are one angle——the turn from the equations of motion (clockwise, $2\pi \sin\varphi$) and the turn from parallel transport (counter-clockwise, Ω) agree modulo 2π. The angle the pendulum turns in a day is $2\pi - \Omega$, that is, 2π minus the solid angle of the cap** (Section 3). **Fourth, the angle does not depend on the pendulum's period——with periods of 2 s and 16 s the turn is 3.603887 rad in both cases, differing by less than 10^{-5} rad** (Section 2). **Fifth, one full turn takes $1/\sin\varphi$ days——1.743447 sidereal days at latitude 35, 1.328040 at 48.85, 1.000000 at the pole, and no turn at the equator** (Section 4). **Sixth, the separator is whether the day is counted in sidereal or in solar days——the turn is tied to the 86164.0905 seconds in which the Earth turns once against the stars; per solar day of 86400 seconds it becomes 207.052861 degrees at 35 degrees** (Section 4). **Keywords**: Foucault pendulum, holonomy, parallel transport, solid angle, sidereal day, Coriolis force.

Scope of this paper (scope note): **No new theorem or law is claimed——that the plane of a Foucault pendulum turns by $2\pi \sin\varphi$ in one sidereal day, and that this angle is the holonomy of parallel transport on the sphere (the enclosed area), are both known (Foucault 1851).No real pendulum is claimed——measured values from Paris or elsewhere are not cited; the latitude 48.85 degrees is a number placed in the computation.Only the linear model is used——precession from finite amplitude (the ellipse), centrifugal force, air resistance, friction at the suspension and the shape of the Earth are not included. Putting a number on the range of small angles is the matter of Paper 396.Geodetic precession is not treated——precession from general relativity is not included.Relation to earlier papers**: **Paper 33 showed that the Wigner angle is an area in velocity space, and that a Foucault pendulum changing direction on a sphere sits, as a theorem, in the same place——here the pendulum's side is counted in three ways.Paper 91 showed that curvature is intrinsic——here that curvature appears in one day, as a change of direction measurable by moving within the sphere alone.Paper 67 read 4π as $2\pi\chi(S^2)$——here the part of that 4π belonging to the cap above the latitude circle, $2\pi(1-\sin\varphi)$, is what is left over from the pendulum's turn.Paper 396 showed that putting a number on a pendulum's "small angle" costs 164.78 seconds a day at ten degrees——here the amplitude is kept small, and it is the direction of the plane of swing, not the period, that is followed through a day.**What is added is **obtaining the same angle from the equations of motion, from parallel transport and from the solid angle of the cap, confirming that the angle does not depend on the period, laying out the days per turn and the turn per hour by latitude**, and **placing the separator on sidereal versus solar days**.

1. Notation and the Question

Setting: **the small swing of a pendulum standing on the ground at latitude φ is written in horizontal coordinates (x, y) (east, north). Only the vertical component $\Omega_z = \Omega_E \sin\varphi$ of the Earth's angular velocity $\Omega_E = 2\pi/86164.0905\ \mathrm{s}^{-1}$ acts as a Coriolis force.**

$$\ddot{x} = -\omega^2 x + 2\Omega_z \dot{y}, \qquad \ddot{y} = -\omega^2 y - 2\Omega_z \dot{x}$$

These linear equations were stepped through one sidereal day with an exact one-step propagator (the matrix exponential) for 0.01 s, and 400 times the azimuth of the point of largest swing within the following period was recorded and joined as the direction of a line.For parallel transport, the latitude circle of the unit sphere was divided into 20000 parts, a tangent vector was carried around by the rotation along the geodesic between neighbouring points, and its change of direction was measured in the east--north frame.The solid angle of the cap was integrated numerically as $\int 2\pi \sin\theta\, d\theta$ up to the colatitude.

Question: **the angle the pendulum turns in a day: what angle is it?**

2. Stepping the Equations of Motion

Computation 2.1: the turn in one sidereal day was obtained at four latitudes (counter-clockwise seen from above counted positive).

latitude (degrees)	turn (rad)	turn (degrees)	$2\pi \sin\varphi$ (rad)	difference (rad)
0	0.000000	0.000000	0.000000	0
35	−3.603887	−206.487516	3.603887	-2.04×10^{-8}
48.85	−4.731172	−271.076195	4.731172	-4.63×10^{-8}
90	−6.283185	−359.999994	6.283185	-1.08×10^{-7}

> **Observation 2.2**
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> stepping the equations of motion gives $2\pi \sin\varphi$.In the northern hemisphere the turn is clockwise, one full turn per day at the pole and none at the equator——with a period of 2 s or of 16 s the turn is 3.603887 rad at latitude 35; the pendulum's period does not enter.

3. Parallel Transport and the Solid Angle of the Cap

Computation 3.1: the change of direction after one circuit of parallel transport along the latitude circle, and the solid angle of the cap above it, were obtained.

latitude (degrees)	change of direction by transport (rad)	solid angle of the cap (numerical, sr)	$2\pi(1-\sin\varphi)$	$2\pi - \Omega$
0	0.000000	6.283185	6.283185	0.000000
35	2.679298	2.679298	2.679298	3.603887
48.85	1.552013	1.552013	1.552013	4.731172
90	0.000000	0.000000	0.000000	6.283185

Observation 3.2

this is the core. The three are one angle.A tangent vector carried once around the latitude circle turns counter-clockwise by the solid angle Ω of the cap above the circle——the plane of swing from the equations of motion turns clockwise by $2\pi \sin\varphi$. Since $2\pi \sin\varphi = 2\pi - \Omega$, the two are the same angle modulo 2π. The plane of swing is parallel-transported relative to the ground, and when the ground returns after one circuit of the latitude circle in a day, its direction is off from the ground by the solid angle of the cap.

4. Days and Time

Computation 4.1: **the sidereal days per full turn $1/\sin\varphi$, the turn per solar day of 86400 s, and the turn per hour were obtained.**

latitude (degrees)	sidereal days per turn	turn per solar day (degrees)	turn per hour (degrees)
35	1.743447	207.052861	8.627203
48.85	1.328040	271.818380	11.325766
90	1.000000	360.985647	15.041069

Observation 4.2

the separator is whether the day is counted in sidereal or in solar days.The pendulum completes its turns against the 86164.0905 seconds in which the ground turns once against the stars, and per solar day of 86400 seconds the turn at 35 degrees becomes 207.052861 degrees——at the pole it turns exactly once per sidereal day and 360.985647 degrees per solar day. Whether the day is counted by the Sun or by the stars decides the angle per day.

5. The Separator and Earlier Papers

> **Observation 5.1**
>
> **placed among the earlier papers.Paper 33 showed that the Wigner angle is an area in velocity space and sits in the same place, as a theorem, as the Foucault pendulum——here the pendulum's side was counted in three ways. Paper 91 showed that curvature is intrinsic; here the change of direction measurable by parallel transport within the sphere alone appears in one day. Paper 67 read 4π as $2\pi\chi(S^2)$; here the cap's solid angle $2\pi(1-\sin\varphi)$ is what is left over from the turn. Paper 396 put a number on a pendulum's small angle; here the amplitude was kept small and only the direction of the plane of swing was followed.**

> **Observation 5.2**
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> **what is not said.Records of real pendulums, precession from finite amplitude, centrifugal force, air resistance, friction at the suspension, the shape of the Earth, and precession from general relativity are not treated——what is shown is that, in the linear model, the equations of motion, parallel transport and the solid angle of the cap return one and the same angle.**

6. Census and Closing

1. $\ddot{x} = -\omega^2 x + 2\Omega_z \dot{y}$ (Section 1): **the linear Coriolis model.**
2. **206.487516 degrees at latitude 35** (Computation 2.1): **within 10^{-7} rad of $2\pi\sin\varphi$.**
3. **359.999994 degrees at the pole, 0 at the equator** (Computation 2.1): **the two ends.**
4. **3.603887 rad with a period of 2 s or 16 s** (Observation 2.2): **the period does not enter.**
5. **2.679298 rad by parallel transport** (Computation 3.1): **equal to the cap's solid angle.**
6. $2\pi - \Omega = 2\pi\sin\varphi$ (Observation 3.2): **the core.**
7. **1.743447 sidereal days per turn** (Computation 4.1): **$1/\sin\varphi$.**
8. **11.325766 degrees per hour** (Computation 4.1): **at latitude 48.85.**
9. **207.052861 degrees per solar day** (Observation 4.2): **206.487516 per sidereal day.**
10. **sidereal, or solar** (Observation 4.2): **the separator; Papers 33, 91, 67, 396.**

Closing: **The angle a Foucault pendulum turns in a day is 2π minus the solid angle of the sky above the latitude circle.Stepping the Coriolis equations of motion through one sidereal day, the plane of swing turned clockwise by**

206.487516 degrees at latitude 35; a tangent vector carried around the latitude circle by parallel transport turned counter-clockwise by the cap's solid angle 2.679298 rad; and the two were one angle modulo 2π——$2\pi \sin\varphi$ is what remains of 2π after the cap's solid angle $2\pi(1-\sin\varphi)$ is taken away.The turn does not depend on the pendulum's period, one full turn takes $1/\sin\varphi$ days, and whether the day is counted in sidereal or solar days decided the angle per day——standing on the ground, the pendulum measures the area of a cap of sky.Placed among the earlier papers ——the pendulum's side, which Paper 33 named as the same place as the Wigner angle, was counted in three ways, and the intrinsic curvature of Paper 91 and the $2\pi\chi(S^2)$ of Paper 67 were shown to appear as one day's turn.To be honest——only the linear model was used, with neither the precession of finite amplitude nor centrifugal force nor friction included.

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