Light Clocks Across and Along the Motion Tick at γ and γ², and Only a 1/γ Contraction Reconciles Them

— a light clock set across the motion ticks at 2γ by Pythagoras alone, while one set along the motion ticks at $2\gamma^2$ unless its arm contracts — the separator is whether the path of the light is perpendicular or parallel to the motion — [Paper 904]

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— From thinking to syncing. —

Abstract

That a moving clock runs slow and that a moving rod contracts are easily taken as two separate facts.This paper sets a clock made of light bouncing between two mirrors across the motion and along it, counts each tick from the geometry of the light path alone, and asks what is needed for the two to agree.No new theorem or law is claimed. First, the transverse light clock ticks at γ by Pythagoras alone——the half tick solves $t = \sqrt{1 + (\beta t)^2}$, and the tick is 2.309401 at $\beta = 0.5$, $\mathbf{4.588315}$ at 0.9 and 14.177624 at 0.99, that is, 2γ. The triangle's vertical side $1/\gamma$ and horizontal side β have a sum of squares of 1.000000: the same triangle as the outline in Paper 901 (Section 2). **Second, the longitudinal light clock ticks at γ^2 unless it contracts——stepping the round trip against a receding and an approaching mirror gives ticks of 2.666667, $\mathbf{10.526316}$ and 100.502513, that is, $2\gamma^2$** (Section 2). **Third, and this is the core. The only contraction that reconciles them is $1/\gamma$——searching by bisection for the arm length at which the longitudinal tick equals the transverse tick gives 0.866025, $\mathbf{0.435890}$ and 0.141067, within 10^{-9} of $1/\gamma$ for all four values of β** (Section 2). **Fourth, with contraction, the orientation of the arm disappears——with the arm at 0, 30, 60 and 90 degrees and contracted by $1/\gamma$, every round trip is 4.588315; uncontracted, they are 10.526316, $\mathbf{9.400301}$, 6.594718 and 4.588315, changing with orientation** (Section 3). **Fifth, without contraction, the fringes move——in a model with 11 m arms, wavelength 0.5 µm and $\beta = 10^{-4}$, the difference between the uncontracted longitudinal and transverse round trips is 1.100000×10^{-7} m, the fringe shift on a 90-degree turn is $\mathbf{0.440000}$, and with contraction by $1/\gamma$ it is 0** (Section 4). **Sixth, the separator is whether the path of the light is perpendicular or parallel to the motion——perpendicular, the tick is set by the hypotenuse of a triangle alone; parallel, by the sum of a receding and an approaching leg. For the two to keep the same time, the parallel arm can only contract by $1/\gamma$** (Section 5). **Keywords**: light clock, time dilation, Lorentz contraction, round-trip light, Pythagorean theorem, fringe shift.

Scope of this paper (scope note): **No new theorem or law is claimed——that time dilation follows from a transverse light clock, that the round trips along two arms fail to agree unless the longitudinal arm contracts by $1/\gamma$ (Michelson and Morley 1887, FitzGerald 1889), and that with contraction the round-trip time does not depend on the arm's orientation (Einstein 1905) are all known.No real apparatus is claimed——the arm length of 11 m, the wavelength of 0.5 µm and $\beta = 10^{-4}$ are model values, not the values or results of any historical experiment.The one-way speed of light is not used——every tick is built from round trips of light alone.Why the contraction occurs is not treated——the contraction appears only as the condition for the two ticks to agree, and its cause is not entered.Relation to earlier papers**: **Paper 24 showed that the one-way speed of light cannot be measured and that the round trip is c under any synchronisation convention——here two clocks built from round trips alone are compared.Paper 901 showed that a moving sphere keeps a circular outline because the $\sqrt{1+\beta^2}$ of light delay and the $\sqrt{1-\beta^2}$ of contraction cancel as the sum of squares $\beta^2 + 1/\gamma^2 = 1$——here the same triangle appears as the path of light in the transverse clock.Paper 12 read $E^2 = (mc^2)^2 + (pc)^2$ in the form of Pythagoras——here the triangle of the light path gives γ in the form of Pythagoras.Paper 123 showed that what differs between the twins is not a lagging clock but the length of a worldline——here a single clock's tick is counted as the length of a light path in the moving frame.Paper 83 showed from Galileo's relativity principle alone that there are only two relativities——here, within one of them, the condition for two light clocks to agree is counted.**What is added is **obtaining the transverse and longitudinal ticks by bisection rather than by formula**, **searching for the reconciling contraction and showing that only $1/\gamma$ exists**, **laying out contracted and uncontracted arms at several orientations**, **giving the fringe shift with model values**, and **placing the separator on perpendicular versus parallel**.

1. Notation and the Question

Setting: **a clock in which light ($c = 1$) bounces between two mirrors 1 apart is carried at speed β.In the transverse clock (mirrors set across the motion) the mirror advances βt during a half tick t, so the light path is a right triangle with hypotenuse t, vertical side 1 and horizontal side βt.In the longitudinal clock (mirrors set along the motion, arm length f) the outgoing light chases a receding mirror and the returning light meets an approaching one.**

$$t = \sqrt{1 + (\beta t)^2}, \qquad t_1 = \frac{f}{1-\beta}, \quad t_2 = \frac{f}{1+\beta}$$

Neither tick was taken from a closed formula; the times at which light and mirror coincide were solved by bisection.The arm length f at which the longitudinal tick equals the transverse tick was also found by bisection.For an arm set at angle θ in the rest frame, only its component along the motion was multiplied by f, and the round trip was solved from two quadratics.

Question: **do a light clock set across the motion and one set along it keep the same time?**

2. Transverse and Longitudinal Ticks

Computation 2.1: for four speeds, the transverse tick, the uncontracted longitudinal tick and the contraction needed to reconcile them were obtained.

β	transverse tick	γ	longitudinal tick (uncontracted)	needed contraction	$1/\gamma$
0.5	2.309401	1.154701	2.666667	0.866025	0.866025
0.9	4.588315	2.294157	10.526316	0.435890	0.435890
0.99	14.177624	7.088812	100.502513	0.141067	0.141067
10^{-4}	2.000000010	1.000000005	2.000000020	0.999999995	0.999999995

> **Observation 2.2**
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> the transverse light clock ticks at γ by Pythagoras alone.From $t^2 = 1 + \beta^2 t^2$ the half tick is $t = \gamma$, and the tick is 2γ——scaling the triangle to hypotenuse 1 gives vertical side $1/\gamma$ and horizontal side β, with a sum of squares of 1.000000. The triangle that restored the outline of a moving sphere to a circle in Paper 901 is here the path of the light.

> **Observation 2.3**
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> this is the core. The only contraction that reconciles them is $1/\gamma$.The uncontracted longitudinal tick is $2\gamma^2$, longer than the transverse tick by γ (2.294157 times at $\beta = 0.9$)——searching for the arm length at which the longitudinal tick equals the transverse tick gave a value within 10^{-9} of $1/\gamma$ for all four values of β. For the two clocks to keep the same time, the arm along the motion can only contract by $1/\gamma$.

3. The Orientation of the Arm

Computation 3.1: at $\beta = 0.9$, with the arm set at rest-frame angle θ, the round-trip time was obtained with the arm contracted by $1/\gamma$ and uncontracted.

θ (degrees)	arm contracted by $1/\gamma$	uncontracted arm	$2\gamma^2\sqrt{1-\beta^2\sin^2\theta}$
0	4.588315	10.526316	10.526316
30	4.588315	9.400301	9.400301
60	4.588315	6.594718	6.594718
90	4.588315	4.588315	4.588315

Observation 3.2

with contraction, the orientation of the arm disappears.The round trip along an arm contracted by $1/\gamma$ is $2\gamma = 4.588315$ in every orientation——along an uncontracted arm it is $2\gamma^2\sqrt{1-\beta^2\sin^2\theta}$, and the values solved from the two quadratics agreed with the closed form within 10^{-12}. Independence of orientation appears only once the contraction is included.

4. The Fringe Shift

Computation 4.1: with 11 m arms, wavelength 0.5 μm and $\beta = 10^{-4}$ (model values), the difference between the uncontracted longitudinal and transverse round trips and the fringe shift on turning the apparatus by 90 degrees were obtained.The difference is 1.100000×10^{-7} m and the fringe shift is 0.440000, agreeing with the approximation $2L\beta^2/\lambda = 0.440000$. With contraction by $1/\gamma$ the shift is 0.

Observation 4.2

without contraction, the fringes move.At $\beta = 10^{-4}$, $\gamma - 1$ is of the order of 5×10^{-9} and invisible in a clock's tick, yet over a round trip along 11 m arms it appears as a difference close to half a wavelength——if the fringes do not move, the longitudinal arm has contracted by $1/\gamma$.

5. The Separator

Observation 5.1

the separator is whether the path of the light is perpendicular or parallel to the motion.Perpendicular, the tick is set by the hypotenuse of a triangle alone, and the arm length does not enter——parallel, the tick is set by the sum of the receding and approaching legs, and the arm length enters as it is. The demand that the two keep the same time fixes the parallel arm's length at $1/\gamma$.

Observation 5.2

placed among the earlier papers.Paper 24 showed that the one-way speed of light cannot be measured and only the round trip is c——here two clocks built from round trips alone were compared. Paper 901 showed that light delay and contraction cancel as a sum of squares; here the same triangle is the light path of the transverse clock. Paper 12 read $E^2 = (mc^2)^2 + (pc)^2$ in the form of Pythagoras; here the triangle of the light path gives γ. Paper 123 showed that the twins differ by the length of a worldline; here a single clock's tick was counted as the length of a light path.

Observation 5.3

what is not said.Real apparatus, the speed of the Earth, the values of historical experiments, the one-way speed of light, and why the contraction occurs are not treated——what is shown is that, counting the ticks of two light clocks from geometry alone, the only reconciling contraction is $1/\gamma$.

6. Census and Closing

1. $t = \sqrt{1 + (\beta t)^2}$ (Section 1): **the transverse light clock.**
2. **transverse tick 4.588315** (Computation 2.1): **2γ at $\beta = 0.9$.**
3. **longitudinal tick 10.526316** (Computation 2.1): **$2\gamma^2$, uncontracted.**
4. **needed contraction 0.435890** (Observation 2.3): **the core; $1/\gamma$.**
5. **sum of squares 1.000000** (Observation 2.2): **the same triangle as Paper 901.**
6. **4.588315 in every orientation when contracted** (Computation 3.1): **four orientations.**
7. **9.400301 at 30 degrees when uncontracted** (Computation 3.1): **$2\gamma^2\sqrt{1 - \beta^2 \sin^2\theta}$.**

8. **fringe shift 0.440000** (Computation 4.1): **0 with contraction.**
9. **$\gamma = 1.000000005$ at $\beta = 10^{-4}$** (Computation 2.1): **invisible in the tick, visible in the fringes.**
10. **perpendicular, or parallel** (Observation 5.1): **the separator; Papers 24, 901, 12, 123, 83.**

Closing: **A transverse light clock ticks at γ by Pythagoras alone; a longitudinal one ticks at γ^2 unless it contracts, and the only contraction that reconciles them is $1/\gamma$.At $\beta = 0.9$ the transverse tick was 4.588315 and the uncontracted longitudinal tick 10.526316, and the contraction found by bisection to reconcile them was $0.435890 = 1/\gamma$——the triangle that sets the transverse tick is the $\beta^2 + 1/\gamma^2 = 1$ with which Paper 901 restored the outline of a moving sphere to a circle.Contracted by $1/\gamma$, the arm's orientation disappears and every round trip is 4.588315; uncontracted, the model arms shift the fringes by 0.440000——fringes that do not move and clocks that run slow are tied together by one and the same contraction.Placed among the earlier papers——two clocks were built from the round-trip light of Paper 24, and their ticks were read with the triangle of Paper 901 and the Pythagorean form of Paper 12.To be honest——the apparatus and the speed are model values, and why the contraction occurs was not entered.**

References

1. Yamagishi, Y. (2026). *The One-Way Speed of Light Cannot Be Measured* [Paper 24]. Zenodo. 10.5281/zenodo.22090261
2. Yamagishi, Y. (2026). *A Moving Sphere Does Not Look Contracted: Light Delay and Contraction Cancel as a Sum of Squares* [Paper 901]. Zenodo. 10.5281/zenodo.22798673
3. Yamagishi, Y. (2026). *From Boundary Energy to Volume Energy* [Paper 12]. Zenodo. 10.5281/zenodo.21404703
4. Yamagishi, Y. (2026). *What Differs Between the Twins Is Not a Lagging Clock but the Length of a Worldline* [Paper 123]. Zenodo. 10.5281/zenodo.22055231
5. Yamagishi, Y. (2026). *Why Does a "Maximum Speed" Exist?* [Paper 83]. Zenodo. 10.5281/zenodo.21741190
6. Michelson, A. A., & Morley, E. W. (1887). On the relative motion of the Earth and the luminiferous ether. *American Journal of Science*.
7. FitzGerald, G. F. (1889). The ether and the Earth's atmosphere. *Science*.
8. Einstein, A. (1905). Zur Elektrodynamik bewegter Körper. *Annalen der Physik*.
9. Taylor, E. F., & Wheeler, J. A. (1992). *Spacetime Physics* (2nd ed.). W. H. Freeman.

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