Cosmic Expansion Follows Exactly from a Newtonian Shell for Dust, but for Light the Deceleration Is Twice the Shell's

— stepping the edge shell of a uniform ball with $\ddot{r} = -GM/r^2$ reproduces Friedmann's dust solution when the shell's energy is 0, and the $8\pi/3$ of the critical density is the $1/2$ of kinetic energy times the $4\pi/3$ of a ball's volume — the separator is whether the contents have pressure — [Paper 907]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— *From thinking to syncing.* —

Abstract

Friedmann's equation for cosmic expansion is usually taken to be an equation of general relativity.This paper moves the edge shell of a uniform ball of matter by Newtonian gravity alone and counts how far that motion agrees with Friedmann's equation and where it departs.No new theorem or law is claimed. First, for dust, the shell's motion is Friedmann's solution itself——with the shell's energy 0, its radius at $t = 1$, 10 and 100 is 1.842016, 6.349604 and 28.356413, agreeing with the dust solution $(1 + 3H_0 t/2)^{2/3}$ to the order of 10^{-14} (Section 2). **Second, the critical density is the density at which the shell's energy is 0——with energy 0, Ω is always 1.000000000; otherwise $\Omega - 1 = -2E/\dot{r}^2$ holds within 10^{-9} ($\Omega = 1.769581$ at $t = 2$ for a closing shell). The curvature term reads as the shell's energy** (Section 3). **Third, $8\pi/3$ is the $1/2$ of kinetic energy and the $4\pi/3$ of the volume of a ball——the same $8\pi/3 = 8.377580$ appears as $\int \sin^2\theta d\Omega$ in Paper 905, but at $d = 4$ the two split into 9.869604 and 14.804407: they are different numbers** (Section 3). **Fourth, and this is the core. In a universe full of light, the deceleration is off by a factor of two——for contents with pressure $p = \rho c^2/3$, the relativistic deceleration $-(4\pi G/3)(\rho + 3p/c^2)$ is 2.000000 times the shell's $-(4\pi G/3)\rho$, and at $t = 10$ the shell universe has expanded 1.838737 times as far as the relativistic one** (Section 4). **Fifth, the factor of departure is $1 + 3w$——with w the ratio of pressure to density, it is 1 for dust, 2 for light, 0 at $w = -1/3$ and -2 at $w = -1$; only pressureless dust matches the shell's 1** (Section 4). **Sixth, the separator is whether the contents have pressure——without pressure, weight is mass alone and the Newtonian shell suffices. With pressure, pressure too has weight, and the shell does not suffice** (Section 5). **Keywords**: Friedmann equation, Newtonian shell, critical density, pressure, cosmic expansion, equation of state.

Scope of this paper (scope note): **No new theorem or law is claimed——Friedmann's equation (1922), the reproduction of pressureless cosmic expansion by a Newtonian shell (Milne 1934; McCrea and Milne 1934), the support in general relativity by Birkhoff's theorem for ignoring the outer shells, and the $\rho + 3p/c^2$ in the deceleration equation are all known.No real universe is claimed——the counting uses a model with $G = 1$ and $H_0 = 1$, and no measured age or density of the universe.The cosmological constant is not treated——dark energy, the cosmological constant and the geometry of spatial curvature are not treated; the curvature term is only read as energy.Relation to earlier papers**: **Paper 485 showed that the radius of Michell's dark star coincides with the Schwarzschild radius, yet the same Newtonian reasoning gives exactly half the general-relativistic bending of light——here too, a shell calculation that agrees exactly for dust gives half the deceleration in a universe full of light. The pattern is the same, but the factor of two here is counted as the pressure's $1 + 3w$, and whether it shares a root with the factor of two there is not judged.Paper 169 showed that the $GM/(Rc^2)$ built from the Hubble radius and the critical density being exactly 0.5 is an identity ——here that 0.5 is counted as the $1/2$ of kinetic energy in the condition that the shell's energy is 0, equal to $\Omega/2$ at any radius.Paper 779 used Friedmann's equation to count the distances light travels——here it is counted how far that equation itself follows from a Newtonian shell.Paper 12 integrated the surface area of a ball to restore the volume $(4/3)\pi c^3$——here the same $4\pi/3$ enters the coefficient of the critical density as the mass inside the shell.Paper 01 counted the exponent at which the mass added per shell is constant——here it is used that matter outside a shell does not affect its motion.**What is added is **matching the shell's motion numerically against Friedmann's dust solution**, **confirming $\Omega - 1 = -2E/\dot{r}^2$**, **separating the identity of $8\pi/3$ from the $8\pi/3$ of Paper 905 by varying d**, **counting that the deceleration is off by a factor of two in a universe full of light**, and **placing the separator on whether the contents have pressure**.

1. Notation and the Question

Setting: **consider a uniform ball of matter of density ρ and follow the shell at its edge, of radius r. The mass inside is $M = (4\pi/3)\rho r^3$, and matter outside the shell is taken not to affect its motion.The shell's motion and its energy E are**

$$\ddot{r} = -\frac{GM}{r^2}, \qquad \frac{1}{2}\dot{r}^2 - \frac{GM}{r} = E \quad \Longrightarrow \quad H^2 = \frac{8\pi G}{3}\rho + \frac{2E}{r^2}$$

with $H = \dot{r}/r$. Friedmann's equations are $H^2 = (8\pi G/3)\rho - kc^2/a^2$ and $\ddot{a}/a = -(4\pi G/3)(\rho + 3p/c^2)$; the critical density is $\rho_c = 3H^2/8\pi G$ and $\Omega = \rho/\rho_c$.The shell's motion was stepped by the Runge--Kutta method with $G = 1$ and $H_0 = 1$ and matched against closed forms of Friedmann's solutions.

Question: **how far does the equation of cosmic expansion follow from Newtonian gravity alone?**

2. Dust: The Shell and Friedmann's Solution

Computation 2.1: **the shell was released at the speed that makes its energy 0, and its radius and Ω were obtained.**

t	shell radius	dust solution $(1+3t/2)^{2/3}$	Ω	$d\ln a/d\ln t$ from the beginning
1	1.842016	1.842016	1.000000000	0.666667
10	6.349604	6.349604	1.000000000	0.666667
100	28.356413	28.356413	1.000000000	0.666667

Observation 2.2

for dust, the shell's motion is Friedmann's solution itself.The relative difference between the shell's radius and the dust solution is of the order of 10^{-14} even at $t=100$, and the expansion exponent measured from the beginning is $2/3$——for pressureless contents the shell's energy equation has the same form as Friedmann's first equation, and the deceleration equation is the same too.

3. Reading Curvature as Energy

Computation 3.1: **for shells with energy below and above 0, $\Omega-1$ and $-2E/\dot r^2$ were set side by side.**

shell	t	Ω	$\Omega-1$	$-2E/\dot r^2$
closing ($E=-0.095000$)	0.5	1.362751	+0.362751	+0.362751
closing	2	1.769581	+0.769581	+0.769581
opening ($E=+0.105000$)	0.5	0.760052	−0.239948	−0.239948
opening	2	0.634185	−0.365815	−0.365815

Observation 3.2

the critical density is the density at which the shell's energy is 0.$\Omega-1=-2E/\dot r^2$ held within 10^{-9} at all six times, with $\Omega>1$ for negative energy and $\Omega<1$ for positive——the curvature term $-kc^2/a^2$ of Friedmann's equation appears in the shell's equation as $2E/r^2$. $\Omega=1$ is the condition of zero energy, at which the shell just escapes to infinity.

Computation 3.3: **the $8\pi/3$ of $\rho_c=3H^2/8\pi G$ was written in d dimensions as "2×(the ball-volume coefficient S_{d-1}/d)" and set beside Paper 905's "$S_{d-1}\times(1-1/d)$".**

d	$2 \times S_{d-1}/d$	$S_{d-1}(1-1/d)$
2	6.283185	3.141593
3	8.377580	8.377580
4	9.869604	14.804407
5	10.527578	21.055156

Observation 3.4

$8\pi/3$ is the $1/2$ of kinetic energy and the $4\pi/3$ of the volume of a ball.Putting $M = (4\pi/3)\rho r^3$ into $H^2 = 2GM/r^3$ gives $H^2 = 2 \times (4\pi/3)G\rho$, where the 2 comes from $\dot{r}^2/2$ and $4\pi/3 = 4.188790$ from the volume of a ball——Paper 905's $\int \sin^2\theta d\Omega = 8\pi/3$ is the same number only at $d = 3$, splitting into 9.869604 and 14.804407 at $d = 4$: the two are different numbers. Moreover, $GM/(Rv^2)$ (with $v = HR$) for a ball of radius R is 0.150000, 0.500000 and 1.500000 for $\Omega = 0.3$, 1 and 3 at every $R = 0.1$, 1, 10, that is, $\Omega/2$. The 0.5 of Paper 169 is this value at $\Omega = 1$.

4. A Universe Full of Light

Computation 4.1: for contents with pressure $p = \rho c^2/3$ diluting as $\rho \propto a^{-4}$, the expansion was stepped with the relativistic deceleration $-(4\pi G/3)(\rho + 3p/c^2)a$ and with the shell's deceleration $-(4\pi G/3)\rho a$ (pressure given no weight).

	relativistic	Newtonian shell	ratio
$\ddot{a}/a$ at $t = 0$	−1.000000	−0.500000	2.000000
a at $t = 1$	1.732051	1.870829	1.080123
a at $t = 10$	4.582576	8.426150	1.838737

Observation 4.2

this is the core. In a universe full of light, the deceleration is off by a factor of two.The relativistic deceleration is 2.000000 times the Newtonian shell's, and the shell universe, decelerated too little, expands faster, reaching 1.838737 times as far at $t = 10$——the relativistic side agreed with $(1 + 2t)^{1/2}$ within 10^{-8} and kept $\Omega = 1$. What the shell calculation lacks is the weight of pressure, $3p/c^2$.

Computation 4.3: for each ratio $w = p/\rho c^2$, the deceleration factor $1 + 3w$ was obtained: 1.000000 for dust ($w = 0$), 2.000000 for light ($w = 1/3$), 0.000000 at $w = -1/3$ and -2.000000 at $w = -1$.

Observation 4.4

the factor of departure is $1 + 3w$.The Newtonian shell's factor is always 1, and it matches only pressureless dust——at $w = -1/3$ the relativistic deceleration vanishes, and at $w = -1$ it changes sign. A shell calculation that counts only positive mass as weight cannot make the deceleration zero or negative.

5. The Separator

Observation 5.1

the separator is whether the contents have pressure.Without pressure, weight is mass alone and the Newtonian shell's motion is Friedmann's equation itself——with pressure, pressure too enters the weight as $3p/c^2$, and the shell calculation mistakes the deceleration by a factor of $1 + 3w$.

Observation 5.2

placed among the earlier papers.Paper 485 showed that Newtonian reasoning agrees at Michell's radius and gives half the general-relativistic bending of light——here too, a shell calculation that agrees exactly for dust gives half the deceleration in a universe full of light. The pattern is the same, but the factor of two here is the pressure's $1 + 3w$, and whether it shares a root with the factor there is not judged. The 0.5 of Paper 169 is the $\Omega/2$ of zero shell energy, the $4\pi/3$ of Paper 12 enters the coefficient of the critical density as the mass inside the shell, and the shell of Paper 01 was used as a shell unaffected by matter outside.

Observation 5.3

what is not said.The density or age of the real universe, the cosmological constant, dark energy, the geometry of spatial curvature, and the general-relativistic grounds for ignoring the outer shells are not treated——what is shown is that Friedmann's equation follows from a Newtonian shell for dust, and that for contents with pressure the deceleration is off by a factor of $1 + 3w$.

6. Census and Closing

1. $H^2 = (8\pi G/3)\rho + 2E/r^2$ (Section 1): **the shell's energy equation.**
2. $\mathbf{28.356413}$ **at** $t = 100$ (Computation 2.1): **the dust solution to** 10^{-14}**.**
3. $\Omega = 1.000000000$ (Computation 2.1): **always, at zero energy.**
4. $\Omega - 1 = -2E/\dot{r}^2$ (Observation 3.2): **curvature reads as energy.**
5. $\Omega = \mathbf{1.769581}$ **for the closing shell** (Computation 3.1): **at** $t = 2$**.**
6. $8\pi/3 = 2 \times 4\pi/3$ (Observation 3.4): **a different number from Paper 905's at** $d = 4$**.**
7. $GM/(Rv^2) = \Omega/2$ (Observation 3.4): **the 0.5 of Paper 169.**
8. **deceleration ratio** $\mathbf{2.000000}$ (Computation 4.1): **the core; a universe full of light.**
9. $\mathbf{1.838737}$ **times at** $t = 10$ (Computation 4.1): **the shell universe expands faster.**
10. **whether there is pressure** (Observation 5.1): **the separator;** $1 + 3w$**; Papers 485, 169, 779, 12, 01.**

Closing: **The equation of cosmic expansion follows exactly from a Newtonian shell for dust, but in a universe full of light the deceleration is off by a factor of two.Moving the edge shell of a uniform ball by Newtonian gravity, a shell of zero energy agreed with Friedmann's dust solution to the order of** 10^{-14} **up to** $t = 100$**, and** Ω **was always** 1**——the critical density is the density of zero energy at which the shell just escapes, the curvature term reads as the shell's energy, and** $8\pi/3$ **is the** $1/2$ **of kinetic energy times the** $4\pi/3$ **of the volume of a ball.In a universe full of light with pressure, the relativistic deceleration was** 2.000000 **times the shell's, and at** $t = 10$ **the shell universe had expanded** 1.838737 **times as far——the factor of departure is** $1 + 3w$**, and what the shell calculation leaves out is the weight of pressure.Placed among the earlier papers——the pattern of Paper 485, "agreeing at one point and off by two at the next", was found in cosmic expansion, and the** 0.5 **of Paper 169 was read as the** $1/2$ **of the shell's kinetic energy.To be honest——this is a model with** $G = 1$**, with neither the cosmological constant nor dark energy, and the grounds for ignoring the outer shells were not entered.**

References

1. Yamagishi, Y. (2026). *Michell's Radius Was Right, but Not for the Reason It Gave* [Paper 485]. Zenodo. 10.5281/zenodo.22759928
2. Yamagishi, Y. (2026). *Sciama's Famous Coincidence Is an Identity* [Paper 169]. Zenodo. 10.5281/zenodo.22093563
3. Yamagishi, Y. (2026). *Distance Is Speed Times Time Only Where Space Does Not Stretch While the Light Travels* [Paper 779]. Zenodo.
4. Yamagishi, Y. (2026). *From Boundary Energy to Volume Energy* [Paper 12]. Zenodo. 10.5281/zenodo.21404703
5. Yamagishi, Y. (2026). *The Uniqueness Theorem of $n = 2$ for Spherically Symmetric Density Distributions* [Paper 01]. Zenodo. 10.5281/zenodo.21403186
6. Yamagishi, Y. (2026). *The 3 in Larmor's 2/3 and the 3 in Its c^3 Are Both the Space Dimension* [Paper 905]. Zenodo. 10.5281/zenodo.22803759
7. Friedmann, A. (1922). Über die Krümmung des Raumes. *Zeitschrift für Physik*.
8. Milne, E. A. (1934). A Newtonian expanding universe. *Quarterly Journal of Mathematics*.
9. McCrea, W. H., & Milne, E. A. (1934). Newtonian universes and the curvature of space. *Quarterly Journal of Mathematics*.
10. Birkhoff, G. D. (1923). *Relativity and Modern Physics*. Harvard University Press.

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)