The 4/3 of the Electromagnetic Mass Comes Not from a Ball's Volume but from Averaging over Field Directions

— the $4/3$ is the ratio 2 of momentum density to energy density times the average $2/3$ over the directions of the field, becoming $2(1-1/d)$ in d dimensions and agreeing with the coefficient of a ball's volume only at $d=2$ and 3 — the separator is whether mass is counted by energy or by momentum — [Paper 908]

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— From thinking to syncing. —

Abstract

Viewing the electron as a small charged ball, it is easy to assume that the $4/3$ of its electromagnetic mass is the same as the $4/3$ in a ball's volume $4\pi R^3/3$.This paper counts by numerical integration the energy and momentum of the field of a moving charged spherical shell, and looks at where the $4/3$ comes from.No new theorem or law is claimed. First, the field momentum is $(4/3)\gamma\beta U_0$——moving a shell of radius 1 and charge 1 at $\beta=0.5$, the momentum of the outside field is 0.384900179 and its energy 0.625462792, agreeing with the closed forms to the order of 10^{-14} (Section 2). **Second, the mass counted by momentum is $4/3$ of the mass counted by energy——$p/(\gamma\beta U_0)$ is 1.333333 at $\beta=0.1$, 0.5 and 0.9 alike, while $U/(\gamma U_0)$ moves through 1.003333, 1.083333 and 1.270000; the two do not combine into one mass** (Section 2). **Third, and this is the core. The $4/3$ is twice the $2/3$ average over the directions of the field——the momentum density $E\times(v\times E)/4\pi$ is $vE^2\sin^2\theta/4\pi$, and the 2 from its ratio to the energy density $E^2/8\pi$ times the spherical average $2/3$ of $\sin^2\theta$ makes $4/3$. In d dimensions it is $2(1-1/d)$: 1.000000, 1.333333, 1.500000, 1.600000 for $d=2$ to 5** (Section 3). **Fourth, it is the same number as the $4/3$ of a ball's volume only in three dimensions or fewer——the coefficient of a ball's volume divided by a power of π is 1, $4/3$, $1/2$, $8/15$, $1/6$ for $d=2$ to 6; it agrees with the ratio of electromagnetic mass only at $d=2$ and 3, departing by 3.000000 times at $d=4$ and 5 and by 10 times at $d=6$** (Section 3). **Fifth, the $2/3$ is the same average as the $2/3$ in Larmor's formula——the $2/3$ of Paper 905 is also $\langle\sin^2\theta\rangle$: both are the square of a cross product with a fixed direction averaged over the sphere** (Section 4). **Sixth, the separator is whether mass is counted by energy or by momentum——counted by energy it is U_0, counted by momentum $(4/3)U_0$, and the gap comes from the average over the directions of the field** (Section 5). **Keywords**: electromagnetic mass, 4/3 problem, field momentum, Poincaré stress, angular average, classical electron.

Scope of this paper (scope note): No new theorem or law is claimed——that the field momentum of a charged spherical shell is $(4/3)\gamma\beta U_0$ and its energy $\gamma U_0(1+\beta^2/3)$, that energy and momentum do not form a single four-vector (Abraham 1903), that adding non-electromagnetic stresses makes them consistent (Poincaré 1906), and the ordering of this treatment (Rohrlich 1960) are all known.Poincaré stresses are not computed——how the stresses fill the gap of $1/3$ is stated as known and not counted.No real electron is claimed——this is a model shell of radius 1 and charge 1, and the electron's actual radius or mass and the treatment in quantum electrodynamics are not entered.Relation to earlier papers: Paper 905 showed that the $2/3$ in Larmor's formula is the $d=3$ case of the spherical average $1-1/d$ of the angular pattern $\sin^2\theta$——the $2/3$ here is the same $\langle\sin^2\theta\rangle$: both are the square of a cross product with a fixed direction (there the acceleration, here the velocity) averaged over the sphere. Since two things met in different places come from the same average, they are written as sharing a root in the sense of Paper 300.Paper 12 read $E=mc^2=4\pi c^2$ in the form of the surface area of a ball and integrated the area to restore the volume $(4/3)\pi c^3$——here the same number $4/3$ appears in the electromagnetic mass, and it is counted that its source is not a ball's volume but the average over the directions of the field.Paper 240 showed that "mass" is seven things——here it is counted that the mass by energy and the mass by momentum differ by $4/3$ in the field of a single charged shell.Paper 300 showed that whether two things share a root is decidable——here, by varying d, the $4/3$ of electromagnetic mass is judged to have a different root from the $4/3$ of a ball's volume and the same root as Larmor's $2/3$.What is added is integrating the field of a moving charged shell numerically on a three-dimensional grid and matching the closed forms, splitting the $4/3$ into the ratio 2 and the average $2/3$, laying it beside the coefficient of a ball's volume for each d to show they agree only in three dimensions or fewer, and placing the separator on energy versus momentum.

1. Notation and the Question

Setting: in Gaussian units with $c=1$, take a charged spherical shell of radius $R=1$ and charge $q=1$. At rest the energy of the outside field is $U_0=q^2/2R=0.5$.When the shell moves in the x direction at speed β, the field in the laboratory frame keeps the parallel component of the rest-frame field $E'=qr'/r'^3$ and multiplies the perpendicular components by γ, with $B=\beta\times E$.

$$U=\int\frac{E^2+B^2}{8\pi}\,dV,\qquad p_x=\int\frac{(E\times B)_x}{4\pi}\,dV,\qquad dV=\frac{dV'}{\gamma}$$

A three-dimensional grid was placed in the rest-frame coordinates (r',θ',φ') (400 Gauss--Legendre points in $r'=R/u$, 200 in θ', 24 in φ'), the laboratory E and B were built, and the integrals were evaluated numerically.The closed forms compared are $U=\gamma U_0(1+\beta^2/3)$ and $p=(4/3)\gamma\beta U_0$.

Question: where does the $4/3$ of electromagnetic mass come from?

2. Field Energy and Momentum

Computation 2.1: for three speeds, the field energy and momentum were integrated numerically.

β	γ	field energy (numerical)	$\gamma U_0(1+\beta^2/3)$	field momentum (numerical)	$(4/3)\gamma\beta U_0$
0.1	1.005038	0.504193971	0.504193971	0.067002521	0.067002521
0.5	1.154701	0.625462792	0.625462792	0.384900179	0.384900179
0.9	2.294157	1.456789910	1.456789910	1.376494403	1.376494403

> **Observation 2.2**
>
> the mass counted by momentum is $4/3$ of the mass counted by energy.The rest field energy is 0.500000000 on the grid too, and the numerical integrals of the moving field agreed with the closed forms to the order of 10^{-14}——$p/(\gamma\beta U_0)$ is 1.333333 at all three speeds, while $U/(\gamma U_0)$ moves through 1.003333, 1.083333 and 1.270000. If field energy and momentum formed a single four-vector, $p/(\gamma\beta)$ and U/γ would both be U_0, but at $\beta = 0.5$ they are 0.666667 and 0.541667.

3. The Content of the $4/3$

Computation 3.1: averaging the momentum density $E \times (v \times E) = vE^2 - E(E\cdot v)$ over the sphere gives $vE^2\langle\sin^2\theta\rangle$, so $p = (v/c^2)2\langle\sin^2\theta\rangle U$. $\langle\sin^2\theta\rangle$ was integrated numerically with the measure of the d-dimensional sphere and set beside the coefficient of the ball volume $V_d = S_{d-1}R^d/d$ divided by $\pi^{\lfloor d/2\rfloor}$.

d	$\langle\sin^2\theta\rangle$	ratio of electromagnetic mass $2\langle\sin^2\theta\rangle$	coefficient of ball volume	ratio
2	0.500000	1.000000	1.000000	1.000000
3	0.666667	1.333333	1.333333	1.000000
4	0.750000	1.500000	0.500000	3.000000
5	0.800000	1.600000	0.533333	3.000000
6	(5/6)	1.666667	0.166667	10.000000

Observation 3.2

this is the core. The $4/3$ is twice the $2/3$ average over the directions of the field.The 2 is the ratio of the momentum density's $1/4\pi$ to the energy density's $1/8\pi$, and $2/3$ is the spherical average of $\sin^2\theta$; in d dimensions the ratio becomes $2(1-1/d)$——ignoring direction the ratio would be 2; the component of the field along the velocity carries no momentum, so $\langle\sin^2\theta\rangle$ multiplies it. The coefficient of a ball's volume agrees with the ratio of electromagnetic mass only at $d=2$ and 3, departing by 3.000000 times at $d=4$ and 5 and by 10 times at $d=6$. The two $4/3$s are different quantities that happen to be the same number only in three dimensions or fewer. At $d=3$ the ratio of $4\pi/3=4.188790$ to $4/3=1.333333$ is π.

4. Larmor's 2/3

Observation 4.1

the $2/3$ is the same average as the $2/3$ in Larmor's formula.Paper 905 showed that the average of the angular pattern $\sin^2\theta$ of radiation from an accelerated charge is $1-1/d$, and that its $d=3$ value $2/3$ is Larmor's coefficient——the $\sin^2\theta$ here comes from the square of the cross product of velocity and electric field. Both are the square of a cross product with a fixed direction (there the acceleration, here the velocity) averaged over the sphere, two things met in different places coming from the same average. In the sense of Paper 300, they are written as sharing a root.

5. The Separator

Observation 5.1

the separator is whether mass is counted by energy or by momentum.Dividing the rest field energy by c^2 gives U_0; dividing the momentum of the moving field by the speed gives $(4/3)U_0$——that $4/3$ does not come from a ball's volume but from the coefficient 2 of momentum density and the $2/3$ average over the directions of the field. That non-electromagnetic stresses fill the gap of $1/3$ between the two masses is known.

Observation 5.2

placed among the earlier papers.Paper 12 read $E = mc^2$ in the form of a ball's surface area and restored the volume $(4/3)\pi c^3$——here the same number $4/3$ appears in the electromagnetic mass, but its source is different and the two split at $d = 4$. Paper 240 showed that "mass" is seven things; here, in one charged shell, the mass by energy and the mass by momentum differed by $4/3$. With the $2/3$ of Larmor in Paper 905 it shares a root, coming from the same average.

Observation 5.3

what is not said.The content of Poincaré stresses, the actual radius and mass of the electron, and the treatment of mass in quantum electrodynamics are not treated——what is shown is that the $4/3$ of the field momentum of a charged shell is twice the $2/3$ average over the directions of the field, and that it is the same number as the $4/3$ of a ball's volume only in three dimensions or fewer.

6. Census and Closing

1. $p_x = \int (E \times B)_x / 4\pi dV$ (Section 1): **field momentum.**
2. **momentum 0.384900179 at $\beta = 0.5$** (Computation 2.1): $(4/3)\gamma\beta U_0$**.**
3. **energy 0.625462792 at $\beta = 0.5$** (Computation 2.1): $\gamma U_0(1 + \beta^2/3)$**.**
4. $p/(\gamma\beta U_0) = 1.333333$ (Observation 2.2): **independent of β.**
5. 0.666667 **against** 0.541667 (Observation 2.2): **not a single four-vector.**
6. $4/3 = 2 \times 2/3$ (Observation 3.2): **the core.**
7. $2(1 - 1/d)$ **in d dimensions** (Computation 3.1): 1**,** $4/3$**,** $3/2$**,** $8/5$**.**
8. 3.000000 **times at $d = 4$** (Computation 3.1): **splits from the ball-volume coefficient.**
9. **the same average as Larmor's $2/3$** (Observation 4.1): **a shared root.**
10. **energy, or momentum** (Observation 5.1): **the separator; Papers 905, 12, 240, 300.**

Closing: **The $4/3$ in the electromagnetic mass of the electron is not the $4/3$ of a ball's volume but twice the $2/3$ average over the directions of the field.Integrating the field of a moving charged shell numerically, the momentum was $(4/3)\gamma\beta U_0$ and the energy $\gamma U_0(1 + \beta^2/3)$, with $p/(\gamma\beta U_0)$ equal to 1.333333 at every speed——the mass by momentum is $4/3$ of the mass by energy, and the two do not combine into one mass.That $4/3$ is the ratio 2 of momentum density to energy density times the spherical average $2/3$ of $\sin^2\theta$, becoming $2(1 - 1/d)$ in d**

dimensions; it agreed with the coefficient of a ball's volume only at $d = 2$ and 3 and departed by 3.000000 times at $d = 4$——the two $4/3$s that looked like one number have different roots, and the $2/3$ side shares its root with Larmor's formula.Placed among the earlier papers——the $4/3$ of electromagnetic mass was set between the volume's $4/3$ of Paper 12 and Larmor's $2/3$ of Paper 905.To be honest——this is a model of radius 1 and charge 1, entering neither the content of Poincaré stresses nor quantum electrodynamics.

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