

Curvature Is Intrinsic

— Riemann, and the Language for Not Conflating the Corpus’s Many “Curvatures” — [Paper 91]

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September 2026 (2nd Edition)

— *From thinking to syncing.* —

Abstract

The **headwater** of the geometric-physics mainstream in which this corpus stands is Bernhard Riemann. His revolution, in one line: **curvature is intrinsic**. The bending of a surface or space is not a property of its embedding into some outer, higher-dimensional space, but is determined by the space’s **own metric** alone. Gauss’s Theorema Egregium (the Gaussian curvature is invariant under isometry — determined by the first fundamental form alone) was generalized by Riemann in 1854 to n dimensions, giving the **Riemann curvature tensor** — a quantity built from the metric and its derivatives (the Christoffel symbols) alone. Intrinsic means: **curvature can be detected from within, without stepping outside** (holonomy of parallel transport; geodesic deviation; the angle sum of a triangle). This paper uses Riemann’s language as a **tool for sorting the corpus’s several “curvatures”**. The corpus features several curvatures — spacetime curvature (general relativity), the **constant velocity-space curvature** $K = -1/c^2$ (Paper 60), and Gauss–Bonnet, which integrates the Gaussian curvature of the 2-sphere to 4π (Paper 67). Riemann **unifies** “what curvature is” into one intrinsic concept, but that does not mean that **every space’s curvature is the same quantity**. Each is an intrinsic quantity of a different manifold, and **although they share the word “curvature”, they are distinct roots** (Papers 48 and 58). **What is shared is the concept (intrinsicness); the instances are separate, one by one.**

Scope of this paper (scope note): This paper is a **structural interpretation** (Layer 2) of established mathematics (Gauss’s Theorema Egregium, Riemannian geometry, the Riemann curvature tensor). Known: the Theorema Egregium (Gauss 1827), the Riemann curvature tensor and intrinsic curvature (Riemann 1854), and the Gauss–Bonnet theorem. The paper contains no new mathematics and no prediction, and does not extend Riemannian geometry. It does not infer identity of root from coincidence of word or value (Papers 48 and 58). “Universe OS” and “matter is curvature” are Layer-3 non-science lenses, not claims of mechanism. Relation to earlier papers: the curvature of velocity space comes from Paper 60, and the Gauss–Bonnet reading of the sphere’s 4π from Paper 67. **What this paper adds** is to apply Riemann’s language to the “curvatures” scattered through the corpus — spacetime, velocity space, sphere — and to sort them, **under one intrinsic concept, as distinct roots living on different manifolds.**

1. Introduction: The Headwater of the Mainstream, and the Polyseme “Curvature”

This corpus has placed itself in the **geometric-physics mainstream** running Riemann → Clifford → Einstein → Weyl → Cartan → Hopf → ... At its headwater stands Bernhard Riemann. The deepest blow he left is the insight that **curvature is intrinsic**.

This insight matters doubly for the corpus. First, all of the corpus’s geometric readings are Riemann’s descendants. Second — and this is the paper’s practical point — the word “curvature” appears in the corpus in **several different places**: spacetime curvature, velocity-space curvature, the sphere’s curvature. Riemann’s language gives the tool to speak of them **with one intrinsic concept while not conflating them**.

2. From the Theorema Egregium to Riemann Curvature: Determined by the Metric Alone

Standard Fact 2.1 (Theorema Egregium → Riemann)

Gauss’s **Theorema Egregium** (1827): the Gaussian curvature K of a surface is invariant under isometry and is determined by the **first fundamental form (the metric) alone** — bend the surface however you like (without stretching) and K does not change. Riemann (1854) generalized this to n -dimensional manifolds and, from the metric g_{ij} and its derivatives, built the **Riemann curvature tensor**

$$R^i_{jkl} = \partial_k \Gamma^i_{lj} - \partial_l \Gamma^i_{kj} + \Gamma^i_{km} \Gamma^m_{lj} - \Gamma^i_{lm} \Gamma^m_{kj}$$

(the Γ are the Christoffel symbols built from the metric). Curvature is defined **intrinsically, with no need for any embedding into an outer space**.

The core here is that curvature is **not “bending as seen from outside”**. A cylinder has Gaussian curvature zero — just a rolled-up plane, intrinsically flat. A sphere has $K > 0$, which a dweller on the sphere can know without stepping outside. Curvature is **a fact internal to the space itself**.

3. What “Intrinsic” Means: Knowing Curvature Without Stepping Outside

Intrinsicness is concrete. In a curved space — **parallel-transporting** a vector around a closed loop rotates it from its original direction (holonomy); nearby **geodesics converge or diverge** (geodesic deviation); the **angle sum of a triangle** departs from π . All of these are fixed by **measurements internal to the space alone**. No outer higher-dimensional space, no embedding, is required. What Riemann opened is a world in which “a space can have its own geometry”, and this prepared the road to Einstein’s gravity (the curvature of spacetime itself).

4. Sorting the Corpus’s Several “Curvatures” (Distinct Roots)

In Riemann’s language, line up the curvatures appearing in the corpus. All are intrinsic curvatures, but they live on **different manifolds — distinct roots**.

Which curvature	Manifold	Intrinsic content	Papers
Spacetime curvature	4D spacetime	Ricci / Riemann tensor (gravity)	(general relativity)
Velocity-space curvature	velocity space	constant negative $K = -1/c^2$	Paper 60
Sphere curvature	S^2	$\int_{S^2} K dA = 2\pi\chi = 4\pi$	Paper 67

Observation 4.1 (One concept, distinct-root instances)

Riemann unifies “what curvature is” into **one intrinsic concept**. But that does not mean that **these curvatures are all the same quantity**. The velocity space’s $K = -1/c^2$ (Paper 60), the spacetime curvature (gravity), and the sphere’s K (Paper 67) are each **intrinsic quantities of different manifolds, distinct roots** that merely share the word “curvature” (Papers 48 and 58). What is shared is the **concept (intrinsicness); the instances are separate, one by one**. Riemann’s language is needed to handle this “one concept, several distinct roots”.

5. Connection to the Corpus: As the Headwater of the Mainstream

Riemann is the **headwater** of the geometric-physics lineage in which the corpus stands (Riemann → Clifford → Einstein → Weyl → Cartan → Hopf → emergent gravity). All of the corpus’s geometric readings — seeing velocity space as a curved space (60), reading 4π as the integral of the sphere’s intrinsic curvature (67), reading gravity as spacetime curvature — are descendants of Riemann’s single blow, “curvature is intrinsic”. The contribution of this paper is not a new theorem but to apply **Riemann’s language to the corpus’s several curvatures and to sort the distinct roots under one concept**.

The author's personal reflection (Layer 3; labelled non-scientific / not a claim of mechanism)

At the close of his 1854 lecture, Riemann left the ground of the metric of space as a question for physics: unless the reality underlying space forms a discrete manifold, **the ground of its metric relations must be sought outside, in binding forces acting upon it**. Curvature can be measured from within, but **what determines** that bending is a question outside geometry — that is the distinction. This seed passed to Clifford's "matter is a bending of space" and to Wheeler's geometrodynamics, and in the corpus's lens (metaphor) becomes the picture that "the Universe OS has no outer scaffold (no embedding) and renders the world from intrinsic geometry". But **this carries the non-science label**. "Matter = curvature" and "OS" are pictures that carry intuition, not mechanisms or predictions. **What is certain here is the standard-geometry point that curvature is intrinsic**. That one mathematician at the headwater taught us that "a space has its own geometry", I record with quiet respect.

6. Conclusion

Riemann's revolution lay in one point: **curvature is intrinsic** — the bending of a space is fixed not by an outer embedding but by the space's own metric alone. The Riemann curvature tensor, generalizing the Theorema Egregium to n dimensions, gave a language to speak of a space's bending from measurements inside, without stepping out. This opened the geometric-physics mainstream, including Einstein's gravity.

This language is also practical for the corpus. The several "curvatures" appearing in it — spacetime, velocity space (60), sphere (67) — **can be spoken of with one intrinsic concept but are distinct roots living on different manifolds**. One concept, distinct instances. What this paper did is to apply the language of Riemann, the headwater of the mainstream, to the corpus's several curvatures and to sort them as distinct roots. A space has its own geometry — to that headwater, we quietly return one address.

Revision Record

2nd edition (September 2026): brought the format into line with the current template (the positioning and scope are gathered in the scope note directly after the abstract, and the section "Scope (what is shown, and what is not)" is removed). Corrected the summary of the close of Riemann's lecture in the personal reflection to match the original (Riemann says that for a continuous manifold the ground of the metric relations must be sought outside, in binding forces acting upon it). Corrected a sentence in the English abstract that could be read as "sharing the word makes them distinct roots". Narrowed the reference for the Gauss–Bonnet reading to Paper 67. Self-citations now carry, one per entry, their actual titles and DOIs, and external references are given as author, year, title and journal.

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