The 1/2 of Kinetic Energy Is One Over the Power 2, and the 1 of Light and the 1 of a Rolling Hoop Have Different Roots

— the ratio of kinetic energy to momentum times speed equals the phase velocity over the group velocity, and is one over the power of momentum in the energy — the separator is whether the momentum divided by is the canonical one or the translational one — [Paper 913]

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— From thinking to syncing. —

Abstract

Paper 18 wrote kinetic energy as $2\pi v^2$ with $m = 4\pi$. Momentum times speed is then $4\pi v^2$, and kinetic energy is exactly half of it.This paper counts what the ratio of kinetic energy to momentum times speed measures, using power-law dispersions, three waves, a relativistic particle and rolling bodies, and separates where the ratio's value 1 comes from in two cases.No new theorem or law is claimed. First, the ratio is the phase velocity over the group velocity, and one over the power of momentum in the energy——taking the energy as momentum to the power n, the ratio is 2, 1, 0.666666667, 0.5 and 0.333333333 for $n = 0.5$, 1, 1.5, 2 and 3, matching the reciprocal and the phase over group velocity within 6.9×10^{-11} (Section 2). **Second, a slow matter wave gives $1/2$, light gives 1, and a deep-water wave gives 2——within 1.7×10^{-10} at three wavenumbers. For a matter wave with $m = 4\pi$, kinetic energy is 6.283185 (2π) times the squared speed, and the 2 of the deep-water wave is the ratio of phase to group velocity confirmed in Paper 515** (Section 2). **Third, as the speed rises, the ratio moves from $1/2$ to 1 as $\gamma/(1+\gamma)$——for a particle with $m = 4\pi$ it is 0.535898385 at $\beta = 0.5$ and 0.876372452 at 0.99, and the power of momentum in the energy is $1 + 1/\gamma$ (1.866025404 at $\beta = 0.5$). The ratio of the kinetic parts of the Hamiltonian and the Lagrangian is exactly γ, and their equality in the slow limit is where the $1/2$ comes from** (Section 3). **Fourth, and this is the core, the 1 of a rolling hoop comes from a different root than the 1 of light——dividing by translational momentum gives 1.000000 for the hoop, but dividing by canonical momentum gives 0.500000 for all five shapes, and the power of translational momentum in the energy stays 2.000000 for all five. With $m = 4\pi$ the hoop's kinetic energy is 12.566371 (4π) times the squared speed, and its canonical momentum is 25.132741 (8π) times the speed** (Section 4). **Fifth, the separator is whether the momentum divided by is the canonical one or the translational one——dividing by canonical momentum, the ratio is one over the power, and the 1 of light comes from the energy having become the first power of momentum. Dividing by translational momentum, the ratio rises by the energy that went into rotation, and that is where the hoop's 1 comes from** (Section 5). **Keywords**: kinetic energy, phase velocity, group velocity, canonical momentum, Legendre transform, rolling rigid body.

Scope of this paper (scope note): **No new theorem or law is claimed——Euler's theorem on homogeneous functions, the distinction between phase and group velocity (Brillouin), the Legendre transform making momentum times speed the sum of the Hamiltonian and the Lagrangian, the relativistic relation between energy and momentum, and the kinetic energy of a rolling rigid body are all standard.A rolling hoop is not said to be the same as light——the ratio is shown to reach the same 1, and the two origins are counted as different.The phase velocity including rest energy is not entered——phase velocity here is kinetic energy over momentum, and the ratio including rest energy is only printed as $1/\beta^2$.No claim about quantum mechanics is made——the dispersion of a matter wave is used as a model for counting the ratio.Rolling friction and slipping are not treated——only ideal rigid bodies rolling without slipping are treated.Mass is not said to be a solid angle——$m = 4\pi$ is a choice of units, and, as in the second edition of Paper 60, the chosen 4π and the 4π of solid angle are different things.Relation to earlier papers**: **Paper 18 wrote kinetic energy as $2\pi v^2$ with $m = 4\pi$——here that 2π is counted as half of momentum times speed, $4\pi v^2$, and the halving as coming from the energy being the square of momentum.Paper 12 read $E = mc^2$ with $m = 4\pi$ as the sphere-area form $4\pi c^2$——here the kinetic energy of a rolling hoop, which takes the same form $4\pi v^2$, is looked at again from its canonical momentum $8\pi v$.Paper 210 showed that four quantities claim the name "speed of light"——here the ratio of phase to group velocity is counted as the ratio of kinetic energy to momentum times speed.Paper 515 confirmed at three wavenumbers that the phase velocity of a deep-water wave is exactly 2 times the group velocity——here that 2 is placed in a row as one over the energy's being momentum to the power $1/2$.Paper 507 showed the point of tangency of the Legendre transform——here momentum times speed as the sum of the Hamiltonian and the Lagrangian is used, and the equality of the two for a slow free particle is counted as the origin of the $1/2$.Paper 240 showed that "mass" is seven things, divided by what it is measured as resistance to——here dividing by canonical or translational momentum is shown to make a rolling hoop's ratio either $1/2$ or 1.**What is added: **counting the ratio of kinetic energy to momentum times speed as phase over group velocity and as one over the power, showing that relativistically the ratio is $\gamma/(1+\gamma)$ and the power $1 + 1/\gamma$, counting that the kinetic parts of the Hamiltonian and Lagrangian are in ratio γ, with the $1/2$ coming from their equality, showing for five shapes that a rolling hoop's 1 returns to $1/2$ when divided by canonical momentum, and placing the separator on "canonical momentum, or translational momentum".**

1. Notation and the Question

Setting: take kinetic energy K as a function of momentum p, the speed (group velocity) as $v = dK/dp$, and the phase velocity as K/p. The ratio examined is R:

$$R = \frac{K}{pv} = \frac{K/p}{dK/dp} = \frac{1}{n}, \qquad n = \frac{d\ln K}{d\ln p}$$

If $K = ap^n$, Euler's theorem $pdK/dp = nK$ gives $R = 1/n$, and the ratio is also the phase velocity divided by the group velocity.Under the Legendre transform, momentum times speed is the sum of the Hamiltonian H and the Lagrangian L:

$$pv = H + L$$

For a slow free particle $H = L = K = p^2/2m$, so $pv = 2K$, that is, $R = 1/2$. With $m = 4\pi$, $p = 4\pi v$, $pv = 4\pi v^2$ and $K = 2\pi v^2$.The check takes derivatives by central differences, and for rolling bodies divides both by translational momentum mv and by canonical momentum $\partial K/\partial v$.

Question: what does the $1/2$ of kinetic energy count? Do light and a rolling hoop, whose ratio is 1, reach 1 for the same reason?

2. Power-Law Dispersions and Three Waves

Computation 2.1: the ratio for $K = ap^n$ ($a = 0.7$, $p = 1.9$).

n	R	phase velocity K/p	group velocity dK/dp	phase over group
0.5	2.000000000	0.507833375	0.253916688	2.000000000
1	1.000000000	0.700000000	0.700000000	1.000000000
1.5	0.666666667	0.964883413	1.447325119	0.666666667
2	0.500000000	1.330000000	2.660000000	0.500000000
3	0.333333333	2.527000000	7.581000000	0.333333333

The ratio differs from $1/n$ and from phase over group velocity by at most 6.9×10^{-11} across the five values of n.

Computation 2.2: three waves ($\hbar = 1$, wavenumber $k = 0.5$): a slow matter wave $\omega = k^2/2m$ ($m = 4\pi$), light $\omega = ck$ ($c = 1$), and a deep-water wave $\omega = \sqrt{gk}$.

wave	phase velocity	group velocity	phase over group
slow matter wave	0.019894368	0.039788736	0.500000000
light	1.000000000	1.000000000	1.000000000
deep-water wave	4.428690551	2.214345276	2.000000000

The ratios are the same at $k = 2$ and 7, with a largest difference of 1.7×10^{-10} over the three wavenumbers. For the matter wave with $m = 4\pi$, kinetic energy divided by the squared group velocity is $\mathbf{6.283185}$ (2π).

> **Observation 2.3**
>
> the ratio is the phase velocity divided by the group velocity, and one over the power of momentum in the energy.The slow matter wave has energy as the square of momentum and gives $1/2$; light has the first power and gives 1; the deep-water wave has the $1/2$ power and gives 2——the 2 of the deep-water wave confirmed in Paper 515 is one member of this row.

3. Raising the Speed

Setting: with $c = 1$ and $m = 4\pi$, the kinetic energy of a particle is written as a function of momentum in a form free of cancellation:

$$K = \frac{p^2}{\sqrt{p^2 + m^2} + m}, \qquad R = \frac{K}{p\beta} = \frac{\gamma}{1+\gamma}, \qquad n = \frac{d\ln K}{d\ln p} = 1 + \frac{1}{\gamma}$$

Computation 3.1: the ratio and the power as β varies (E is the energy including rest energy).

β	γ	R	n	$E/(p\beta)$
0.001	1.000000500	0.500000125	1.999999500	1000000.000000
0.1	1.005037815	0.501256289	1.994987437	100.000000
0.5	1.154700538	0.535898385	1.866025404	4.000000
0.9	2.294157339	0.696432229	1.435889895	1.234568
0.99	7.088812050	0.876372452	1.141067360	1.020304
0.9999	70.712445952	0.986055419	1.014141782	1.000200

The ratio differs from $1/n$, and the group velocity dK/dp from β, by at most 2.8×10^{-10}; the closed forms $\gamma/(1+\gamma)$ and $1 + 1/\gamma$ are matched within 1.1×10^{-16} and 4.1×10^{-10}.The ratio including rest energy, $E/(p\beta)$, matched $1/\beta^2$.

Computation 3.2: the kinetic parts with rest energy removed, $H_{\text{kin}} = (\gamma - 1)m$ and $L_{\text{kin}} = (1 - 1/\gamma)m$.

β	H_{kin}	L_{kin}	$H_{\text{kin}}/L_{\text{kin}}$
0.5	1.944024300	1.683574429	1.154701
0.9	16.262860751	7.088816655	2.294157
0.99	76.514268823	10.793665890	7.088812

For six values of β, $p\beta = H_{\text{kin}} + L_{\text{kin}}$ holds within 10^{-9}, and for a slow free particle ($m = 4\pi$, $v = 1.3$), $pv = 21.237166338 = 2K$.

> **Observation 3.3**
>
> as the speed rises, the ratio moves continuously from $1/2$ to 1 as $\gamma/(1+\gamma)$.Because the kinetic parts of the Hamiltonian and the Lagrangian are in ratio exactly γ, the ratio is $H/(H+L) = \gamma/(1+\gamma)$——their equality in the slow limit is where the $1/2$ comes from, and the 1 of light sits where the power has come all the way down to 1.

4. The Rolling Hoop

Setting: a body of mass m and radius r rolls without slipping at speed v. With moment of inertia $I = kmr^2$:

$$K = \frac{1}{2}m(1+k)v^2, \qquad p_{\text{trans}} = mv, \qquad P = \frac{\partial K}{\partial v} = m(1+k)v$$

Computation 4.1: five shapes with $m = 4\pi$.

shape	k	K/v^2	$K/(p_{\text{trans}}v)$	P/v	$K/(Pv)$	power of translational momentum
sliding only	0	6.283185	0.500000	12.566371	0.500000	2.000000
solid sphere	0.4	8.796459	0.700000	17.592919	0.500000	2.000000
disc	0.5	9.424778	0.750000	18.849556	0.500000	2.000000
thin spherical shell	2/3	10.471976	0.833333	20.943951	0.500000	2.000000
hoop	1	12.566371	1.000000	25.132741	0.500000	2.000000

Differentiating the energy by translational momentum and dividing by the speed gives 1.000000, 1.400000, 1.500000, 1.666667 and 2.000000 ($= 1 + k$) for the five shapes, not the speed.

> **Observation 4.2**
>
> **the 1 of a rolling hoop is not the 1 of a power.Dividing by translational momentum, the ratio is $(1 + k)/2$ and the hoop gives 1.000000, but the energy remains the square of translational momentum, with power 2.000000——differentiating the energy by translational momentum does not give the speed, so the ratio is not one over the power. Dividing by canonical momentum returns all five shapes to 0.500000. With $m = 4\pi$ the hoop has $K = 4\pi v^2$, and its canonical momentum is $8\pi v$, $\mathbf{25.132741}$ times the speed.**

5. The Separator

> **Observation 5.1**
>
> **the separator is whether the momentum divided by is the canonical one or the translational one.Dividing by canonical momentum, the ratio is one over the power of momentum in the energy, equal to the phase velocity over the group velocity——the 1 of light is because the power became 1, the 2 of a deep-water wave because the power is $1/2$, and the $1/2$ of a slow body because the power is 2. Dividing by translational momentum, the ratio rises by the energy that went into rotation, and that is where the hoop's 1 comes from.**

> **Observation 5.2**
>
> **what is not said.That a rolling hoop is the same as light, the phase velocity including rest energy, quantum mechanics, rolling friction and slipping, and mass being a solid angle are not treated——what is shown is that the ratio of kinetic energy to momentum times speed is one over the power, and that the same 1 has two different origins.**

6. Roll Call and Closing

1. $R = 1/n$ (Section 1): **phase over group velocity.**
2. **difference 6.9×10^{-11} over five n** (Computation 2.1): **one over the power.**
3. **matter wave 0.5, light 1, deep-water wave 2** (Computation 2.2): **Paper 515.**

4. $K/v^2 = \mathbf{6.283185}$ (Computation 2.2): **the 2π of Paper 18.**

5. $\mathbf{0.535898385}$ **at** $\beta = 0.5$ (Computation 3.1): $\gamma/(1+\gamma)$**.**

6. $H_{\text{kin}}/L_{\text{kin}} = \gamma$ (Computation 3.2): **the origin of the $1/2$.**

7. **hoop 1.000000, and 0.500000 with canonical momentum** (Computation 4.1): **the core.**

8. **canonical momentum $\mathbf{25.132741}v$** (Observation 4.2): 8π**.**

9. **canonical, or translational** (Observation 5.1): **the separator; Papers 18, 12, 210, 515, 507, 240.**

Closing: **The $1/2$ of kinetic energy is one over the power 2, and the 1 of light and the 1 of a rolling hoop have different roots.The ratio of kinetic energy to momentum times speed equals the phase velocity over the group velocity and is one over the power of momentum in the energy: $1/2$ for a slow matter wave, 1 for light, and 2 for a deep-water wave——raising the speed moves the ratio from $1/2$ to 1 as $\gamma/(1+\gamma)$, which was 0.535898385 at $\beta = 0.5$. The kinetic parts of the Hamiltonian and the Lagrangian are in ratio γ, and their equality in the slow limit is where the $1/2$ comes from.A rolling hoop gives 1 when divided by translational momentum, but all five shapes return to $1/2$ when divided by canonical momentum, with the power of the energy remaining 2——the separator is whether the momentum divided by is canonical or translational.Placed among the earlier papers——the $2\pi v^2$ of Paper 18 is read as half of momentum times speed, $4\pi v^2$; the rolling hoop's $4\pi v^2$, which takes the same form as the $4\pi c^2$ of Paper 12, is looked at again from its canonical momentum $8\pi v$; and the 2 of Paper 515 is placed in the row of reciprocal powers.To be honest——a rolling hoop is not said to be the same as light. The same number 1 has two different origins, and no further.**

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