

With Two Shadows the Sun Drops Out of Earth's Circumference, Leaving the Angle Reading and the North-South Distance

— the difference of the noon zenith angles at two towns is the difference in latitude itself, and the Sun's declination does not enter — the separator is whether the Sun's zenith angle is set to **0** at one town or measured at both — [Paper 914]

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September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/914>

— *From thinking to syncing.* —

Abstract

Earth's circumference can be found from noon shadows measured on the same day at two towns separated north to south, using the difference of the angles and the distance between the towns. This paper counts what moves the result of this method and what does not, with representative coordinates (Alexandria and Aswan) on a spherical Earth. No new theorem or law is claimed. First, with two shadows the Sun's direction drops out: the noon zenith angles at the two towns are **7.76** and **0.65** degrees, and their difference, **7.11** degrees, is the difference in latitude itself. Moving the Sun's declination by **1** degree leaves the difference unchanged (difference **0**) (Section 2). Second, with only one shadow the Sun's direction enters the angle: assuming the Sun overhead at the southern town, the angle is read as **1.091421** times the latitude difference and the circumference comes out **0.916237** times as large. The shadow then cast by a rod at the southern town is **0.011345** times its height (Section 2). Third, only the north-south component of the distance counts: the great-circle distance between the towns is **843.208** km and its north-south component **790.597** km; using the great-circle distance makes the circumference **1.066546** times as large, while the north-south component returns the sphere's circumference of **40030.229** km (Section 3). Fourth, the remaining angle reading and the unit of length also move the circumference: reading the angle **0.1** degree too small makes the circumference **1.4085** percent larger, and the circumference from $1/50$ of a circle and 5000 stadia differs by a factor of **1.174603** depending on whether a stadion is taken as 157.5 m or 185 m (Section 3). Fifth, the separator is whether the Sun's zenith angle is set to **0** at one town or measured at both: measured at both, the Sun's direction cancels in the subtraction, and only the angle reading and the distance move the circumference (Section 4).

Keywords: Eratosthenes, Earth's circumference, zenith angle, solar declination, great-circle distance, error propagation.

Scope of this paper (scope note): No new theorem or law is claimed: finding Earth's circumference from the shadows at two towns, the noon zenith angle as the difference between latitude and the Sun's declination, and great-circle distance on a sphere are all standard. History is not judged. What Eratosthenes actually measured, which stadion he used, and which value was right are not treated. The two lengths are only set side by side as representative values. The coordinates are representative: Alexandria at 31.20 degrees north, 29.92 degrees east; Aswan at 24.09 degrees north, 32.90 degrees east; the Sun's declination at the solstice 23.44 degrees; Earth's mean radius 6371.0088 km. Earth is taken as a sphere: flattening, atmospheric refraction and the blurring of the shadow by the Sun's apparent size are not included. Relation to earlier papers: Paper 814 showed that the distance to the Moon comes out of an eclipse shadow and a clock alone. Here, the size of Earth needed before that is found from the noon shadow angle and the distance between towns, and the way errors enter is counted. Paper 297 showed that cosmic distance is a ladder of seven rungs of which only the first is measured directly. Here, what matters in measuring the size of Earth, at the foot of that ladder, with two shadows is separated. Paper 340 showed on the same sphere of radius 6371.0088 km that the shortest path and the path of constant bearing are different. Here, the great-circle distance between two towns and the north-south component that enters the circumference are counted as different. What is added: confirming that the Sun's declination drops out of the difference of two shadow angles, finding that one shadow reads the angle as 1.091421 times the latitude difference, finding that the great-circle distance makes the circumference 1.066546 times as large, setting side by side how the angle reading and the unit of length act, and placing the separator on "the zenith angle set to 0 at one town, or measured at both".

1. Notation and the Question

Setting: a northern town A and a southern town S are separated north to south, and at noon on the same day the Sun's zenith angles z_A and z_S are measured from the shadows of rods. With latitude φ and the Sun's declination δ , the noon zenith angles are:

$$z_A = \varphi_A - \delta, \quad z_S = \varphi_S - \delta, \quad z_A - z_S = \varphi_A - \varphi_S$$

With the distance between the towns d and the angle difference θ in degrees, the circumference is $C = 360d/\theta$. The check places representative coordinates for Alexandria (A) and Aswan (S) on a spherical Earth and finds the great-circle distance and the north-south component separately.

Question: in finding the circumference from shadows, what moves the result and what does not?

2. One Shadow and Two Shadows

Computation 2.1: noon zenith angles at the solstice ($\delta = 23.44$ degrees).

town	latitude (deg)	noon zenith angle (deg)
Alexandria	31.20	7.76
Aswan	24.09	0.65
difference	7.11	7.11

The difference of zenith angles is the latitude difference itself, and moving the declination by 1 degree left the difference unchanged (difference 0).

Computation 2.2: assuming the Sun overhead at the southern town (one shadow). Using the northern town's zenith angle of 7.76 degrees directly, the angle is read as **1.091421** times the latitude difference and the circumference comes out 0.916237 times as large. The shadow of a rod at the southern town was 0.011345 times the rod's height.

Observation 2.3

with two shadows, the Sun's direction cancels in the subtraction. With only one shadow, the amount by which the Sun misses the overhead point at the southern town enters the angle directly: that miss remains as a shadow 0.011345 times the height of a rod at the southern town.

3. Distance and the Angle Reading

Computation 3.1: how the distance is taken (latitude difference 7.11 degrees).

distance taken as	distance (km)	circumference (km)
north-south component	790.597	40030.229
great-circle distance	843.208	42694.064

The circumference from the north-south component matched the sphere's circumference to the order of 10^{-11} km, and the great-circle distance made it **1.066546** times as large.

Computation 3.2: the angle reading and the unit of length.

condition	circumference
angle read 0.1 degree too small	1.014085 times (1.4085 percent larger)
angle read 0.5 degree too small	1.074627 times (7.4627 percent larger)
7.2 degrees, 5000 stadia at 157.5 m	39375.0 km (0.983632 of the sphere)
7.2 degrees, 5000 stadia at 185 m	46250.0 km (1.155377 of the sphere)

The circumferences from the two stadia differ by a factor of **1.174603**, and the **7.2** degrees of $1/50$ of a circle is **1.012658** times the **7.11**-degree latitude difference of these coordinates.

Observation 3.3

once two shadows remove the Sun's direction, what moves the circumference is the angle reading and the distance. Only the north-south component of the distance counts, not the route, and **0.1** degree of angle is worth **1.4085** percent of the circumference: the choice of the unit of length moves the circumference far more than that.

4. The Separator

Observation 4.1

the separator is whether the Sun's zenith angle is set to **0** at one town or measured at both. Set to **0** at one town, the difference between the Sun's declination and that town's latitude enters the angle directly, and with these coordinates the angle becomes **1.091421** times as large: measured at both, the declination cancels in the subtraction, leaving the angle reading, the distance as a north-south component, and its unit of length.

Observation 4.2

what is not said. What Eratosthenes actually measured, which stadion is right, flattening and atmospheric refraction are not treated. What is shown is that the difference of two shadows removes the Sun's direction, and that the remaining errors come from the distance and the angle reading.

5. Roll Call and Closing

1. $z_A - z_S = \varphi_A - \varphi_S$ (Section 1): **the declination drops out.**
2. **zenith angles 7.76 and 0.65 degrees, difference 7.11** (Computation 2.1): **the latitude difference.**
3. **one shadow: 1.091421 times** (Computation 2.2): **circumference 0.916237 times.**
4. **great-circle distance: 1.066546 times** (Computation 3.1): **north-south component gives 40030.229 km.**
5. **0.1 degree: 1.4085 percent** (Computation 3.2): **the angle reading.**
6. **stadia: 1.174603 times** (Computation 3.2): **the unit of length.**
7. **zero at one town, or measured at both** (Observation 4.1): **the separator; Papers 814, 297, 340.**

Closing: With two shadows the Sun drops out of Earth's circumference, leaving the angle reading and the north-south distance. The difference of the noon zenith angles at two towns is the latitude difference itself and did not change when the Sun's declination was moved: assuming the Sun overhead at the southern town read the angle as 1.091421 times as large with these coordinates. Only the north-south component of the distance counts, and the great-circle distance made the circumference 1.066546 times as large. A tenth of a degree of angle is worth 1.4085 percent, and the choice of the unit of length is worth more: the separator is whether the Sun's zenith angle is set to 0 at one town or measured at both. Placed among the earlier papers: the size of Earth, which sits before the Moon's distance found by shadow and clock in Paper 814, was counted as the foot of the ladder of Paper 297, on the same sphere as the great circles of Paper 340. To be honest, which length was right is not said. What two shadows remove and what they leave, and no further.

References

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