# A Degree of Latitude Is Longer Near the Pole, and Flattening Read from Two Arcs Magnifies Their Error Over a Hundredfold

— a degree is longer at the pole because the meridian curves more gently there, and flattening is read from a slight difference between two arcs — the separator is whether the length of one arc is read, or the ratio of two arcs minus 1 — [Paper 915]

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*— From thinking to syncing. —*

**Abstract**

**Taking Earth as an ellipsoid of revolution (WGS84), the length of one degree of latitude along the meridian changes with latitude.This paper counts how much that length differs between pole and equator, why it is longer at the pole, and how much an error in an arc counts when flattening is found back from the lengths of two arcs.No new theorem or law is claimed. First, a degree of latitude is longer near the pole——the degree spanning the equator is 110574.304 m and the degree spanning the pole 111693.951 m, longer by 1119.647 m (ratio 1.010126). In steps of 15 degrees, the length grew monotonically with latitude** (Section 2). **Second, it is longer at the pole because the meridian curves more gently there——the meridian radius of curvature is 6335439.327 m at the equator and 6399593.626 m at the pole, matching the closed forms to the order of $10^{-9}$ m. The flatter the pole, the larger the radius of curvature and the longer the arc needed for the same degree of angle** (Section 2). **Third, flattening comes back from two arcs, but an error in an arc counts over a hundredfold——solving from the one-degree arcs at latitudes 1.5 and 66.3 degrees (ratio 1.008471) returns a flattening of 1/298.257224. Measuring the northern arc 0.1 percent too long gives 1/266.698, and the relative change in flattening was 118.335 times the relative error in the arc** (Section 3). **Fourth, the magnification is roughly one over the ratio of the two arcs minus 1——the guide from the ratio 1.008471 is 118.045, close to the measured magnification. The meridian from equator to pole is 10001965.729 m, a ratio of 1.00019657 to ten million metres** (Section 3). **Fifth, the separator is whether the length of one arc is read, or the ratio of two arcs minus 1——the length of one arc reads the size of Earth, and a 0.1 percent error in the arc is only 0.1 percent of the size. Flattening reads the ratio minus 1, so the same error spreads over a hundredfold** (Section 4). **Keywords**: meridian arc, ellipsoid of revolution, flattening, radius of curvature, error magnification, WGS84.

**Scope of this paper (scope note)**: **No new theorem or law is claimed——the meridian radius of curvature of an ellipsoid of revolution, the integral for meridian arc length, and finding flattening from two arcs are all standard.History is not judged——which survey measured what and when, and whether its value was right, are not treated. Latitudes $1.5$ and $66.3$ degrees are representative.Only the ellipsoid is used——the undulation of the geoid, gravity, atmospheric refraction and survey methods are not included. The WGS84 semi-major axis $6378137$ m and flattening $1/298.257223563$ are used.The definition of the metre is not discussed——only one ratio, of the meridian quadrant to ten million metres, is set down.Relation to earlier papers**: **Paper 404, in its scope note, took Earth as a sphere and did not make flattening or the swell of the sea its subject; this paper sits in that seat of flattening** (the earlier occupant is Paper 787, which counted the distance of summits from the centre in the same seat of flattening).**Paper 212 showed that every map must distort something——here, before any map, Earth's own departure from a sphere is counted as it appears in the length of a degree of latitude.Paper 787 showed that on an ellipsoid of revolution the order of summits by distance from the centre and by height can swap——here, what the curvature of the meridian of the same ellipsoid does to the length of a degree and to reading flattening is counted.**What is added: **deriving the $1119.647$ m difference between the polar and equatorial degree from the radius of curvature**, **setting down the length of a degree every $15$ degrees**, **solving flattening back from two arcs by bisection**, **finding that an arc error counts $118.335$ times in flattening**, and **placing the separator on "the length of one arc, or the difference in the ratio of two"**.

## 1. Notation and the Question

**Setting**: **for an ellipsoid of revolution with semi-major axis $a$ and flattening $f$, let $e^2 = f(2 - f)$. The meridian radius of curvature $M$ at latitude $\varphi$ and the meridian arc length $L$ from $\varphi_1$ to $\varphi_2$ are:**

$$M(\varphi) = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \varphi)^{3/2}}, \qquad L = \int_{\varphi_1}^{\varphi_2} M(\varphi)\, d\varphi$$

**At the equator $M = a(1 - e^2)$, and at the pole $M = a/\sqrt{1 - e^2}$.The check sums the arc length by Simpson's rule and solves $e^2$ from the ratio of two arcs by bisection.**

**Question**: **how much does a degree of latitude differ between pole and equator, and how much does an error in an arc count when flattening is read from two arcs?**

## 2. The Length of a Degree of Latitude

**Computation 2.1**: **the one-degree arc spanning each latitude, WGS84.**

| latitude (deg) | one-degree arc (m) |
|---|---|
| 0 | 110574.304 |
| 15 | 110648.721 |
| 30 | 110852.457 |
| 45 | 111131.778 |
| 60 | 111412.273 |
| 75 | 111618.359 |
| 90 | 111693.951 |

The difference between pole and equator is $1119.647$ m and the ratio $1.010126$; checked every $3$ degrees, the one-degree arc grew monotonically with latitude.

Computation 2.2: the meridian radius of curvature is $6335439.327$ m at the equator and $6399593.626$ m at the pole, matching the closed forms $a(1-e^2)$ and $a/\sqrt{1-e^2}$ within $9.3 \times 10^{-10}$ m.

> **Observation 2.3**
>
> a degree is longer at the pole because the meridian curves more gently there.Latitude is measured by the direction of the normal to the meridian, so the arc needed for one degree of angle is proportional to the radius of curvature——at the flattened pole the radius of curvature is large, and the same degree needs a longer arc.

## 3. Reading Flattening from Two Arcs

Computation 3.1: the one-degree arcs spanning latitudes $1.5$ and $66.3$ degrees are $110575.065$ m and $111511.782$ m (difference $936.717$ m, ratio $1.008471$), and the flattening solved from that ratio was $1/298.257224$ (difference from WGS84 $1.14 \times 10^{-11}$).

Computation 3.2: measuring only the northern arc too long.

| error in northern arc | solved flattening | relative change in flattening over relative arc error |
|---|---|---|
| 0.01 percent | 1/294.767 (1.011840 times) | 118.402 |
| 0.10 percent | 1/266.698 (1.118335 times) | 118.335 |

One over the ratio of the two arcs minus 1 was 118.045. The first-order approximation gave $e^2 = 0.006741307$, and bisection 0.006694380.

Computation 3.3: the meridian from equator to pole is 10001965.729 m, a ratio of **1.00019657** to ten million metres.

> **Observation 3.4**
>
> flattening is read from the slight difference of the ratio of two arcs from 1.The ratio is 1.008471, only a little over eight thousandths away from 1——an arc error enters flattening as a fraction of that difference, so the magnification is 118.335, close to one over the difference, 118.045.

## 4. The Separator

> **Observation 4.1**
>
> the separator is whether the length of one arc is read, or the ratio of two arcs minus 1.The length of one arc reads the size of Earth, so a 0.1 percent error in the arc stays 0.1 percent of the size——flattening reads the difference in the ratio, so the same error appears as a flattening 1.118335 times as large.

> **Observation 4.2**
>
> what is not said.Which survey was right, the geoid and gravity, and the history of the definition of the metre are not treated——what is shown is why a degree of latitude is longer at the pole, and how an arc error is magnified when flattening is read from two arcs.

## 5. Roll Call and Closing

1. $M(\varphi)$ and the arc-length integral (Section 1): the radius of curvature.

2. **pole minus equator 1119.647 m** (Computation 2.1): **longer at the pole.**
3. **radii of curvature 6335439.327 m and 6399593.626 m** (Computation 2.2): **gentler curvature at the pole.**
4. **flattening returns to 1/298.257224** (Computation 3.1): **from the ratio of two arcs.**
5. **error magnified 118.335 times** (Computation 3.2): **close to one over the difference, 118.045.**
6. **quadrant ratio 1.00019657** (Computation 3.3): **to ten million metres.**
7. **one arc, or the difference in a ratio** (Observation 4.1): **the separator; the scope note of Paper 404; Papers 212 and 787.**

**Closing**: **A degree of latitude is longer near the pole, and flattening read from two arcs magnifies their error over a hundredfold.On the WGS84 meridian, the degree spanning the pole was 1119.647 m longer than the degree spanning the equator——because the radius of curvature is larger at the pole, where the meridian curves more gently.Flattening comes back from the ratio of two arcs, but the ratio is 1.008471, only slightly different from 1, so a 0.1 percent error in the northern arc made the flattening 1.118335 times as large——the separator is whether the length of one arc is read, or the ratio of two arcs minus 1.Placed among the earlier papers——the flattening left open by Paper 404's sphere was counted before the distortion of maps in Paper 212, on the meridian of the same ellipsoid as Paper 787.To be honest——which survey was right is not said. How differently precise arc length and flattening demand to be, and no further.**

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