

The Metonic Cycle Is a Convergent of Year over Month, and Fitting Whole Days Too Makes the Miss Over Three and a Half Times Larger

— 235 months and 19 years agree as a convergent, but the day misses separately as a third clock — the separator is whether two clocks are fitted or three — [Paper 916]

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— *From thinking to syncing.* —

Abstract

A lunisolar calendar counts months by the phases of the Moon and inserts an occasional leap month to stay with the seasons. This paper confirms how the Metonic cycle, 235 months in 19 years, comes out of the continued fraction of the ratio of year to month, and then counts how the miss changes when the days are made whole. No new theorem or law is claimed. First, $235/19$ is a convergent of the ratio of year to month: the tropical year of 365.24219 days divided by the synodic month of 29.530589 days is 12.368266342, and the convergents of its continued fraction run $12/1, 25/2, 37/3, 99/8, 136/11, 235/19, 4131/334$. A search of all denominators up to 19 also found $235/19$ best (Section 2). Second, 235 months and 19 years differ by only 0.086805 days: the eight-year cycle $99/8$ differs by 1.5908 days, and the Metonic drift accumulates to 1 day in 218.881 years. There are 7 leap months in 19 years (Section 2). Third, fitting the days too makes the miss larger: the difference between 235 months and the nearest whole number of days, 6940, is 0.311585 days, 3.589482 times the miss between year and month. The difference between 19 years and the nearest whole number of days is 0.398390 days (Section 3). Fourth, the Callippic cycle refits the days: setting 76 years and 940 months to 27759 days, four times 6940 less 1, leaves a difference of 0.246340 days from the months and 0.593560 days from the years (Section 3). Fifth, the separator is whether two clocks are fitted or three: with year and month alone, the convergents of the continued fraction choose the best cycle. Adding the day as a third clock imposes the separate condition that the cycle be a whole number of days, which the convergents do not look after (Section 4). **Keywords:** Metonic cycle, synodic month, tropical year, continued fraction, convergent, lunisolar calendar.

Scope of this paper (scope note): No new theorem or law is claimed: that convergents of a continued fraction give best approximations, and the Metonic, Callippic and eight-year cycles, are all standard. The Antikythera Mechanism is not decoded: proposals for its gear tooth counts or how its dials are read are not entered. That the mechanism kept calendar cycles is only mentioned by name. History is not judged, which calendar was used when, and who found the cycles, are not treated. The lengths are mean representative values: a tropical year of 365.24219 days and a synodic month of 29.530589 days are used, without tidal change or month-to-month fluctuation. Relation to earlier papers: Paper 319, in its scope note, did not treat other calendars such as lunar and lunisolar ones; this paper sits in that seat of the lunisolar calendar (no earlier occupant). Paper 539 showed that the tooth counts of Huygens's planetarium are convergents of the continued fraction of the ratio of the periods of Saturn and Earth. Here the same tool is applied to the ratio of year to month, and it is counted that with three clocks the convergents alone do not decide. Paper 812 showed that how easily a number is approximated is decided by whether its partial quotients are bounded. Here a partial quotient of 17 appears in the ratio of year to month, and the convergent just before it, 235/19, fits especially well. Paper 406 showed that seeing the same face is the evidence that the Moon rotates. Here only the synodic month, the period of the Moon's phases, is used. What is added: setting down the continued fraction and convergents of the ratio of year to month, confirming by exhaustive search that 235/19 is best for denominators up to 19, finding that the miss when fitted to whole days is 3.589482 times the year-month miss, counting the Callippic refitting of the days, and placing the separator on "two clocks fitted, or three".

1. Notation and the Question

Setting: with tropical year Y and synodic month M , the ratio $r = Y/M$ is expanded as a continued fraction. A convergent h/k stands for " h months in k years", with drift $hM - kY$ days.

$$r = \frac{Y}{M} = 12 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \dots}}}, \quad \Delta = hM - kY$$

The check uses $Y = 365.24219$ days and $M = 29.530589$ days, compares all fractions with denominators up to 19, and finds the differences from whole numbers of days.

Question: where do 235 months in 19 years come from, and how does the miss change when the days are made whole too?

2. Year and Month

Computation 2.1: the partial quotients of $r = 12.368266342$ were [12; 2, 1, 2, 1, 1, 17, 3, 14]. The drift of year against month for each convergent:

convergent (months/years)	years (days)	months (days)	drift (days)
12/1	365.2422	354.3671	−10.8751
25/2	730.4844	738.2647	+7.7803
37/3	1095.7266	1092.6318	−3.0948
99/8	2921.9375	2923.5283	+1.5908
136/11	4017.6641	4016.1601	−1.5040
235/19	6939.6016	6939.6884	+0.0868
4131/334	121990.8915	121990.8632	−0.0283
12628/1021	372912.2760	372912.2779	+0.0019

Among denominators up to 19, the fraction with smallest $|h - kr|$ was 235/19 (0.002939 months). The Metonic drift is +0.086805 days, accumulating to 1 day in 218.881 years. There are $235 - 12 \times 19 = 7$ leap months.

Observation 2.2

235/19 is a convergent of the ratio of year to month.It is the fraction cut off just before the partial quotient 17 appears, so it fits well for the size of its denominator. The product of denominator and miss, $k|h - kr|$, is 0.055850 for 235/19, an order of magnitude below the other convergents (0.314397 to 0.560227) and close to $1/17 = 0.058824$: with year and month as the only two clocks, the continued fraction chooses the best cycle.

3. Adding the Day

Computation 3.1: differences from whole numbers of days.

compared	difference (days)
235 months and 19 years	+0.086805
6940 days and 235 months	+0.311585
19 years and the nearest whole number of days	+0.398390

The miss when fitted to whole days was **3.589482** times the miss between year and month.

Computation 3.2: the Callippic cycle (76 years, 940 months, 27759 days). 940 months are 27758.753660 days and 76 years 27758.406440 days; the differences from 27759 days are **0.246340** days for the months and 0.593560 days for the years.

Observation 3.3

fitting the days too makes the miss larger than with year and month alone. Year and month agree within 0.086805 days, but the length of that cycle lies 0.311585 days from a whole number of days: the Callippic cycle, taking 1 day from the 27760 days of four Metonic cycles, brings the day miss against the months back to 0.246340 days.

4. The Separator

Observation 4.1

the separator is whether two clocks are fitted or three. With year and month, the convergents of the continued fraction of their ratio choose the best cycle for each denominator, and at 19 years that is 235/19: adding the day as a third clock brings the separate condition that the cycle be a whole number of days, which the convergents do not look at. The miss therefore became 3.589482 times as large.

Observation 4.2

what is not said. The gear teeth of the Antikythera Mechanism, the history of calendars, and changes in the lengths of the year and month are not treated. What is shown is that the Metonic cycle is a convergent of year over month, and that adding the day brings a separate miss.

5. Roll Call and Closing

1. $r = 12.368266342$ (Computation 2.1): year over month.
2. 235/19 is a convergent, best for denominators up to 19 (Computation 2.1): chosen by the continued fraction.
3. year-month drift 0.086805 days (Computation 2.1): 1 day in 218.881 years.
4. difference from whole days **0.311585** (Computation 3.1): **3.589482** times.

5. Callippic cycle **0.246340** days (Computation 3.2): **refitting the days.**
6. **two clocks or three** (Observation 4.1): **the separator; the scope note of Paper 319; Papers 539, 812, 406.**

Closing: The Metonic cycle is a convergent of year over month, and fitting whole days too makes the miss over three and a half times larger. Among the convergents of the ratio 12.368266342 is $235/19$, best for denominators up to 19, with a drift of 0.086805 days. Yet its difference from 6940 days is 0.311585 days, 3.589482 times the year-month miss. The Callippic cycle takes away 1 day and brings the miss against the months back to 0.246340 days. With year and month alone the convergents choose the cycle, but adding the day imposes the separate condition of whole days: the separator is whether two clocks are fitted or three. Placed among the earlier papers: the lunisolar calendar left untreated by Paper 319 was expanded with the same continued fractions as the tooth counts of Paper 539, and the size of the partial quotients of Paper 812 was seen inside the ratio of year to month. To be honest: no mechanism has been decoded. The difficulty of fitting three clocks at once was placed in one number, and no further.

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