

How Hard a Stone Must Be Shaken to Topple Is Set by Shape Alone When Slow, but Larger Stones Need More When the Shaking Is Brief

— under a sustained sideways acceleration the toppling threshold is the ratio of width to height with no size in it, but in one brief cycle of shaking a larger stone tips more slowly — the separator is whether the period of shaking is longer or shorter than the stone’s rocking time — [Paper 917]

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Abstract

A stacked stone, shaken sideways, topples or slides. This paper treats a rectangular stone that does not slip as a single rigid body and counts, for a sustained acceleration and for one cycle of sinusoidal shaking, when the toppling threshold is independent of size and when it is not. No new theorem or law is claimed. First, under a sustained sideways acceleration, toppling or sliding is decided by two ratios: toppling begins when the acceleration exceeds width over height times gravity, sliding when it exceeds the friction coefficient times gravity, and the smaller happens first. Scaling width and height by 10 left the toppling threshold unchanged (difference 0) (Section 2). Second, in one brief cycle of shaking, a larger stone needs stronger shaking to topple: shaking a stone of width over height 0.2 through one sinusoidal cycle of period 0.5 s, the smallest toppling amplitude was $0.26g$ at a height of 0.490 m, $0.30g$ at 0.981 m, $0.44g$ at 1.961 m and $1.03g$ at 3.922 m (Section 3). Third, a larger stone tips more slowly: the $p = \sqrt{3g/4R}$ of the rocking equation fell from 5.424016 to 1.917679 per second over the four sizes, and the ratio of the rocking time $2\pi/p$ to the period of shaking rose from 2.316802 to **6.552905** (Section 3). Fourth, in slow shaking, size disappears and only shape remains: with shaking of period 20 s, stones of height 0.490 m and 3.922 m both toppled at $0.205g$, and neither toppled at $0.19g$. The model kept the energy of free rocking within 9.65×10^{-6} , and a time step four times smaller did not change the verdicts (Section 3). Fifth, the separator is whether the period of shaking is longer or shorter than the stone’s rocking time: if longer, the threshold is set by the ratio of shape alone; if shorter, size enters through the rocking time. The coefficient of restitution at landing, **0.942308**, was also set by shape, not by material (Section 4). **Keywords:** toppling of rigid bodies, sliding, Housner rocking block, coefficient of restitution, size effect, one-cycle pulse.

Scope of this paper (scope note): No new theorem or law is claimed: the balance of a rigid body tipping about a corner, the condition for friction to give way, and the equation of a rocking rigid body with its coefficient of restitution at landing (Housner) are all standard. The seismic safety of walls and buildings is not discussed: the design of real masonry, safety factors and codes are not treated. In the shaking computation the stone does not slip: interlocking between stones, deformation and rocking while sliding are not included. Sliding is treated only in the comparison under sustained acceleration. No real earthquake records are used: only one cycle of sinusoidal shaking is used. The numbers are values of the model. The toppling amplitude is not said to be monotone in amplitude: only the smallest value, searched in steps of $0.05g$ and refined in steps of $0.01g$, is counted. Relation to earlier papers: Paper 454, in its scope note, did not make seismic force its subject and looked only at overturning, not sliding; this paper sits in that seat (no earlier occupant). Paper 355 showed that the coefficient of restitution is not a material constant. Here, the coefficient of restitution at the landing of a rocking stone is used as fixed at 0.942308 by the ratio of width to height alone. Paper 747 showed that once grade enters the denominator, a railway's hauling margin disappears. Here, which of the friction coefficient and the ratio of width to height is smaller is counted as deciding whether a stone slides or topples first under a sideways acceleration. What is added: confirming that the toppling threshold does not depend on size, finding the smallest toppling amplitude under one cycle of shaking for four sizes, setting down the ratio of rocking time to period of shaking, confirming that size disappears in slow shaking, and placing the separator on "the period of shaking longer or shorter than the rocking time".

1. Notation and the Question

Setting: a rectangular stone of width b and height h is a rigid body that does not slip. Let the slenderness angle be $\tan \alpha = b/h$, the distance from the centre to a corner R , the horizontal ground acceleration $a(t)$ and the tilt θ .

$$\ddot{\theta} = -p^2 \left[\sin(\alpha \operatorname{sgn} \theta - \theta) + \frac{a(t)}{g} \cos(\alpha \operatorname{sgn} \theta - \theta) \right], \quad p^2 = \frac{3g}{4R}$$

Tilting begins when $|a| > g \tan \alpha$, toppling when $|\theta| > \alpha$, and at each landing ($\theta = 0$) the angular velocity is multiplied by $r = 1 - \frac{3}{2} \sin^2 \alpha$. The check applies $a(t) = A \sin(2\pi t/T)$ for one cycle, integrates by Runge-Kutta, and after the shaking stops ends the run according to whether enough energy remains to topple.

Question: when is the strength of shaking that topples a stone set by shape alone, and when does size enter?

2. Sustained Sideways Acceleration

Computation 2.1: comparing the toppling threshold b/h with the sliding threshold μ .

width/height	friction coefficient	happens first	threshold
0.2	0.4	topples	0.20g
0.2	0.6	topples	0.20g
0.5	0.4	slides	0.40g
0.5	0.6	topples	0.50g
1.0	0.4	slides	0.40g
1.0	0.6	slides	0.60g

The toppling threshold from the balance of moments about a corner was the same with width and height scaled by 10 and mass by 1000 (difference 0).

Observation 2.2

under a sustained sideways acceleration, toppling or sliding is decided by two ratios, with no size in them. Weight and inertia are both proportional to the same mass, so the scales of mass and length drop out of the balance: whichever of width over height and the friction coefficient is smaller happens first.

3. One Cycle of Shaking

Computation 3.1: width over height 0.2 ($\alpha = 11.3099$ degrees, coefficient of restitution 0.942308), one sinusoidal cycle of period $T = 0.5$ s.

half-diagonal R (m)	height (m)	p (per s)	$2\pi/p$ over T	smallest toppling amplitude
0.25	0.490	5.424016	2.316802	0.26g
0.50	0.981	3.835359	3.276453	0.30g
1.00	1.961	2.712008	4.633604	0.44g
2.00	3.922	1.917679	6.552905	1.03g

The rocking times $2\pi/p$ were 1.158401, 1.638226, 2.316802 and 3.276453 s. Runs cut off without a verdict: 0.

Computation 3.2: checks on the model. In free rocking with no shaking and no restitution ($\theta_0 = \alpha/2$, 5 s), the energy drifted by 9.65×10^{-6} (in units of p^2); with a time step four times smaller, “topples at the smallest amplitude and not 0.01g below” held unchanged for all four sizes. With shaking of period 20 s, both $R = 0.25$ m and 2.00 m toppled at **0.205g**, and neither toppled at 0.19g.

Observation 3.3

in brief shaking, a larger stone needs stronger shaking to topple. With the same shape, a larger stone has a smaller p and a rocking time longer compared with the period of shaking: in slow shaking both sizes toppled at the same 0.205g, coming down near the sustained threshold of 0.2g.

4. The Separator

Observation 4.1

the separator is whether the period of shaking is longer or shorter than the stone’s rocking time. If the period of shaking is long, the shaking acts like a sustained acceleration and the threshold is set by the ratio of width to height alone: if short, size enters through the rocking time $2\pi/p$, and in this model a stone 3.922 m tall needed 1.03g. The coefficient of restitution at landing, 0.942308, was also set by the ratio of width to height alone.

Observation 4.2

what is not said. The seismic safety of walls and buildings, real earthquake records, rocking while sliding, and how toppling amplitudes are ordered are not treated. What is shown is that whether size enters the strength of shaking that topples a stone divides on how the period of shaking compares with the rocking time.

5. Roll Call and Closing

1. the rocking equation, $p^2 = 3g/4R$ (Section 1): Housner.
2. toppling threshold b/h , sliding threshold μ (Computation 2.1): no size in them.
3. 0.26, 0.30, 0.44, 1.03g (Computation 3.1): size enters in brief shaking.

4. $2\pi/p$ over T from 2.316802 to **6.552905** (Computation 3.1): **the rocking time.**
5. **0.205g** at period 20 s (Computation 3.2): **back to shape alone.**
6. **coefficient of restitution 0.942308** (Computation 3.1): **set by shape.**
7. **period of shaking and rocking time** (Observation 4.1): **the separator; the scope note of Paper 454; Papers 355 and 747.**

Closing: How hard a stone must be shaken to topple is set by shape alone when slow, but larger stones need more when the shaking is brief. Under a sustained sideways acceleration, toppling or sliding was decided by the two ratios of width over height and the friction coefficient, and scaling width and height by 10 did not change the threshold: in one cycle of shaking of period 0.5 s, stones of the same shape toppled at 0.26g when 0.490 m tall and at 1.03g when 3.922 m tall. With shaking of period 20 s, both sizes toppled at the same 0.205g, and size disappeared: the separator is whether the period of shaking is longer or shorter than the stone's rocking time. Placed among the earlier papers: the seismic force and sliding left untreated by Paper 454 were separated by the same two ratios as the comparison in Paper 747, and the coefficient of restitution of Paper 355 was used as one set by shape. To be honest: the safety of stone walls is not claimed. When size breaks into a threshold set by shape, and no further.

References

1. Housner, G. W. (1963). The behavior of inverted pendulum structures during earthquakes. *Bulletin of the Seismological Society of America*.
2. Yamagishi, Y. (2026). *The Coefficient of Restitution Is Not a Material Constant — And the Empirical Law Points at Its Own Breaking Speed* [Paper 355]. YOI: <https://www.yamagishiyuuki.com/yoi/355>; DOI: 10.5281/zenodo.22697011
3. Yamagishi, Y. (2026). *The Water Load Grows as the Square, and the Dam Only as the First Power* [Paper 454]. YOI: <https://www.yamagishiyuuki.com/yoi/454>; DOI: 10.5281/zenodo.22732989
4. Yamagishi, Y. (2026). *A Railway's Hauling Margin Is Large Only on the Flat; Once Grade Enters the Denominator, Rubber Tyres Win* [Paper 747]. YOI: <https://www.yamagishiyuuki.com/yoi/747>; DOI: 10.5281/zenodo.23015117
5. Zhang, J., & Makris, N. (2001). Rocking response of free-standing blocks under cycloidal pulses. *Journal of Engineering Mechanics*.

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