

Treating the Leg as a Pendulum Misses the Cadence, but the Top Walking Speed Is Set by Leg Length and Gravity

— the cadence of the leg as a hanging pendulum falls short of ordinary cadence, but inverting the same pendulum gives the speed at which the push on the ground vanishes at the top — the separator is whether the leg is seen as a hanging pendulum or the body as an inverted pendulum on top of it — [Paper 921]

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— *From thinking to syncing.* —

Abstract

In walking, the leg in the air swings forward like a pendulum, and the body passes over the planted leg like an upside-down pendulum. This paper counts these two pendulums for a leg 0.90 m long, and shows that one misses the cadence while the other gives the top speed at which walking can continue. No new theorem or law is claimed. First, the cadence of the hanging pendulum falls short of ordinary cadence: taking half a period as one step, a simple pendulum gives 63.0435 steps per minute and a uniform rod 77.2122, which are **0.573123** and 0.701929 times an ordinary cadence of 110 steps per minute (representative) (Section 2). Second, the same pendulum inverted gives the top walking speed: the speed at which the centripetal acceleration needed at the top of the arc over the leg equals gravity is $\sqrt{gL} = \mathbf{2.970856}$ m/s, where the push on the ground at the top became 0 (Section 3). Third, as speed rises, the push on the ground at the top falls: per body weight it is 0.777928 at 1.4 m/s, 0.291864 at 2.5 m/s and -0.160210 at 3.2 m/s; beyond the top speed the leg would have to pull on the ground (Section 3). Fourth, with leg length, cadence and top speed move in opposite directions: a leg four times as long halved the cadence and doubled the top speed. Ordinary walking at 1.4 m/s has a Froude number of 0.222072, the speed at a walk-to-run Froude number of 0.5 (representative) is 2.100712 m/s, and under lunar gravity the top speed was 1.207477 m/s (Section 4). Fifth, the separator is whether the leg is seen as a hanging pendulum or the body as an inverted pendulum on top of it: the hanging pendulum sets the natural swing rate of the leg, and cadence is faster than that. The inverted pendulum sets the ceiling on the speed at which the body can pass over the leg without lifting off (Section 5).

Keywords: walking, pendulum, inverted pendulum, cadence, Froude number, leg length.

Scope of this paper (scope note): No new theorem or law is claimed: the periods of simple and physical pendulums, the model of walking as an inverted pendulum, and comparing walking and running by the Froude number are all standard. What sets cadence is not claimed: muscle action, energy use and neural control are not treated. It is only counted that the pendulum cadence falls short of ordinary cadence. The values are representative: leg length 0.90 m, ordinary cadence 110 steps per minute, ordinary walking speed 1.4 m/s and a walk-to-run Froude number of 0.5 are representative. The leg is a rigid rod or a point mass: knee bending, ankle action and arm swing are not included. How people move on the Moon is not claimed: only one top speed under changed gravity is set down. Relation to earlier papers: Paper 380 showed that the limit on running speed is not how fast the legs move but how hard the ground is pushed. Here, for walking rather than running, the speed at which the push on the ground becomes 0 at the top is counted as setting the limit. Paper 396 showed that putting a number on a pendulum's small angle makes ten degrees cost 164.78 seconds a day. Here the small-angle period is used as it is, and what the cadence of the leg pendulum gets right and wrong is counted. What is added: comparing the cadences of two pendulums with ordinary cadence, finding the speed at which the push on the ground vanishes at the top of the inverted pendulum, setting down the push at the top for several speeds, varying leg length and gravity, and placing the separator on "hanging pendulum, or inverted pendulum".

1. Notation and the Question

Setting: let the leg length be L and gravity g . Treating the leg in the air as a pendulum with half a period as one step, the cadence (steps per minute) for a simple pendulum and a uniform rod is:

$$n_{\text{simple}} = \frac{60}{\pi\sqrt{L/g}}, \quad n_{\text{rod}} = \frac{60}{\pi\sqrt{2L/3g}}$$

When the body passes over the planted leg at speed v , the centripetal acceleration of the centre of mass at the top of its arc is v^2/L , and the push on the ground at the top per body weight is $1 - v^2/gL$. The speed at which it becomes 0 is $v_{\text{max}} = \sqrt{gL}$, and the Froude number is v^2/gL .

Question: what does the model of the leg as a pendulum get right about walking, and what does it miss?

2. The Leg as a Hanging Pendulum

Computation 2.1: with $L = 0.90$ m, the simple-pendulum cadence was 63.0435 steps per minute and the uniform-rod cadence 77.2122. Their ratios to an ordinary cadence of 110 steps per minute (representative) are **0.573123** and 0.701929.

Observation 2.2

the cadence of the hanging pendulum falls short of ordinary cadence. Even distributing the leg's inertia as a rod reaches only seven tenths: ordinary walking proceeds at a cadence faster than the leg's natural swing.

3. The Body as an Inverted Pendulum

Computation 3.1: with $L = 0.90$ m, the top speed was $\sqrt{gL} = \mathbf{2.970856}$ m/s. The push on the ground at the top per body weight:

speed (m/s)	push on the ground at the top (per body weight)
1.400000	0.777928
2.500000	0.291864
2.970856	0.000000
3.200000	−0.160210

Observation 3.2

the inverted pendulum gives the ceiling on walking speed. Raising the speed lowers the push on the ground at the top, which reaches 0 at 2.970856 m/s: faster than that, keeping the body over the leg would require the leg to pull on the ground, and inverted-pendulum walking cannot continue.

4. Varying Leg Length and Gravity

Computation 4.1: varying leg length.

leg length	simple-pendulum cadence	rod cadence	top speed (m/s)
0.45 m	89.1570	109.1946	2.100712
0.90 m	63.0435	77.2122	2.970856
1.80 m	44.5785	54.5973	4.201425

A leg four times as long halved the cadence and doubled the top speed.

Computation 4.2: ordinary walking at 1.4 m/s has a Froude number of 0.222072, and a Froude number of 0.5 (representative) corresponds to 2.100712 m/s. Under lunar gravity of 1.62 m/s squared the top speed is 1.207477 m/s (0.406441 of Earth's).

Observation 4.3

with leg length, cadence and top speed move in opposite directions. A longer leg swings more slowly by nature but has a higher ceiling on the speed of passing over it: ordinary walking at Froude number 0.222072 is below the ceiling, and the walk-to-run Froude number of 0.5 is also short of the ceiling of 1.

5. The Separator

Observation 5.1

the separator is whether the leg is seen as a hanging pendulum or the body as an inverted pendulum on top of it. The hanging pendulum sets the natural swing rate of the leg, and ordinary cadence is about $1/0.573123$ times faster: the inverted pendulum sets the ceiling $\sqrt{gL}$ on the speed at which the body can pass over the leg without lifting off, fixed by leg length and gravity alone.

Observation 5.2

what is not said. What sets cadence, the action of knee and ankle, the actual reason for changing to running, and how people move on the Moon are not treated. What is shown is that the same pendulum model misses cadence when hanging and gives the ceiling on speed when inverted.

6. Roll Call and Closing

1. $n_{\text{simple}}, n_{\text{rod}}, v_{\text{max}} = \sqrt{gL}$ (Section 1): **two pendulums.**
2. cadences **63.0435** and **77.2122**, ratio **0.573123** (Computation 2.1): **short of cadence.**
3. top speed **2.970856** m/s (Computation 3.1): **push at the top is 0.**
4. **0.777928** at 1.4 m/s, **-0.160210** at 3.2 m/s (Computation 3.1): **beyond the top, a pull.**

5. **four-times leg: half cadence, double top speed** (Computation 4.1): **opposite directions**.
6. **Froude number 0.222072; 1.207477 m/s on the Moon** (Computation 4.2): **below the ceiling**.
7. **hanging, or inverted** (Observation 5.1): **the separator; Papers 380 and 396**.

Closing: Treating the leg as a pendulum misses the cadence, but the top walking speed is set by leg length and gravity. The cadence of a 0.90 m leg as a hanging pendulum was 63.0435 steps per minute, only 0.573123 of ordinary cadence: inverting the same pendulum gave the speed of 2.970856 m/s at which the push on the ground at the top becomes 0, beyond which the leg would have to pull on the ground. A leg four times as long halves the cadence and doubles the top speed, and the two move in opposite directions: the separator is whether the leg is seen as a hanging pendulum or the body as an inverted pendulum on top of it. Placed among the earlier papers: the push on the ground that Paper 380 examined in running was counted at the top of a walking step, and the small-angle period of Paper 396 was applied to the cadence of the leg. To be honest. What sets cadence is not said. What the same model misses and what it gets right, and no further.

References

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