

Brightness Follows the Square of the F-Number When the Half-Angle Is Measured by Its Sine, and in Air the F-Number Cannot Go Below 0.5

— for a lens satisfying the sine condition, image illuminance falls exactly by half per stop in inverse proportion to the square of the f-number, and at f-number **0.5** light arrives from the whole image-side hemisphere — the separator is whether the half-angle of the aperture is measured by its sine or by its tangent — [Paper 923]

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September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/923>

— *From thinking to syncing.* —

Abstract

In photography, changing the f-number by factors of $\sqrt{2}$ is said to change image brightness by halves. Starting from the proportionality of image illuminance to the square of the sine of the image-side half-angle, this paper counts in which way of measuring that halving is exact, how far it misses when the measure is changed, and how small an f-number can be. No new theorem or law is claimed. First, measuring the half-angle by its sine, illuminance falls exactly by half per stop: setting the f-number to half the reciprocal of the sine of the half-angle, illuminance is inversely proportional to the square of the f-number, and over 11 steps of $\sqrt{2}$ the ratio departed from 2 by only 4.4×10^{-16} . The ratio between F1 and F16 is **256.000000** (Section 2). Second, at f-number **0.5** light arrives from the whole image-side hemisphere: the half-angle is then 90 degrees and the illuminance π (3.141593); a smaller f-number would need a sine above 1 and cannot be made in air (Section 2). Third, measuring the half-angle by its tangent misses on the bright side: the ratio of illuminance measured by tangent to that measured by sine is 0.500000 at f-number 0.5, **0.800000** at 1, 0.941176 at 2 and 0.999024 at 16, and the ratio between F1 and F16 was **205.000000** (Section 3). Fourth, illuminance looks not at the solid angle but at the projected solid angle: the ratio of projected solid angle to the solid angle of the exit cone is 0.500000 at f-number 0.5, 0.933013 at 1 and 0.984123 at 2 (Section 4). Fifth, the separator is whether the half-angle of the aperture is measured by its sine or by its tangent: measured by the sine, the square law in the f-number is exact and the lower limit of the f-number is 0.5. Measured by the tangent, the values approach the same on the dark side but depart from the square law the brighter the setting (Section 5). **Keywords:** f-number, image illuminance, sine condition, numerical aperture, projected solid angle, photographic lens.

Scope of this paper (scope note): No new theorem or law is claimed: the proportionality of image illuminance to the squared sine of the image-side half-angle, the sine condition, the relation between working f-number and numerical aperture, and projected solid angle are all standard. Only the axis is treated: fall-off of brightness toward the edge of the frame, vignetting, transmittance and reflection losses are not included. No lens is designed: aberrations are not computed; whether the sine condition is satisfied is only set as two ways of measuring. Real lenses are not discussed: f-number markings of products and which lens is brighter are not treated. Air only: the lower limit is stated for an image-side medium of refractive index 1. Relation to earlier papers: Paper 358 showed that the depth of field is a convention, not a measurement. Here, for brightness, it is counted in which way of measuring the same f-number becomes exact. Paper 757 showed that which of wavelength, numerical aperture and factor buys the resolution decides what the depth of focus pays. Here the same sine as in the numerical aperture is counted as deciding the square law of brightness in the f-number and its lower limit of 0.5. Paper 918 showed that antenna gain is a ratio measuring how the 4π of solid angle is shared out. Here it is counted that image illuminance looks not at solid angle but at projected solid angle, and reaches the hemisphere's projected solid angle π at f-number 0.5. What is added: confirming that illuminance measured by the sine halves exactly per step of $\sqrt{2}$, showing that illuminance reaches π at the lower limit of f-number 0.5, setting down the departures when measured by the tangent, finding the ratio of projected to plain solid angle, and placing the separator on "sine, or tangent".

1. Notation and the Question

Setting: imaging a surface of luminance L , image illuminance can be written with the image-side half-angle θ' as follows (transmittance 1). Two ways of relating the f-number N to the half-angle, by sine and by tangent, are set side by side:

$$E = \pi L \sin^2 \theta', \quad \sin \theta' = \frac{1}{2N} \Rightarrow E = \frac{\pi L}{4N^2}, \quad \tan \theta' = \frac{1}{2N} \Rightarrow E = \frac{\pi L}{1 + 4N^2}$$

Measuring by sine corresponds to a lens satisfying the sine condition, and by tangent to the geometry of a lens with a flat principal plane. The check sets $L = 1$ and varies the f-number from 0.5 to 16 in steps of $\sqrt{2}$.

Question: in which way of measuring is the halving of brightness with the square of the f-number exact, and how small can the f-number be?

2. Measuring by the Sine

Computation 2.1: illuminance and half-angle for each f-number.

f-number	illuminance by sine	illuminance by tangent	tangent over sine	half-angle by sine (deg)	half-angle by tangent (deg)
0.500000	3.141593	1.570796	0.500000	90.0000	45.0000
0.707107	1.570796	1.047198	0.666667	45.0000	35.2644
1.000000	0.785398	0.628319	0.800000	30.0000	26.5651
1.414214	0.392699	0.349066	0.888889	20.7048	19.4712
2.000000	0.196350	0.184800	0.941176	14.4775	14.0362
2.828427	0.098175	0.095200	0.969697	10.1821	10.0250
4.000000	0.049087	0.048332	0.984615	7.1808	7.1250
5.656854	0.024544	0.024353	0.992248	5.0709	5.0512
8.000000	0.012272	0.012224	0.996109	3.5833	3.5763
11.313708	0.006136	0.006124	0.998051	2.5330	2.5305
16.000000	0.003068	0.003065	0.999024	1.7908	1.7899

Illuminance measured by sine departed from a ratio of 2 per step of $\sqrt{2}$ by at most 4.4×10^{-16} , and the ratio between F1 and F16 was **256.000000**. The illuminance at f-number 0.5 matched π with difference 0.

Observation 2.2

measured by the sine, the square law in the f-number is exact and the lower limit is 0.5. At f-number 0.5 the half-angle becomes 90 degrees and light arrives from the whole image-side hemisphere: a smaller f-number asks for a sine above 1 and cannot be made on the image side in air.

3. Measuring by the Tangent

Computation 3.1: the ratio of illuminance by tangent to illuminance by sine is $4N^2/(1 + 4N^2)$, matching **0.800000** at f-number 1 with difference 0. The ratio of illuminance between F1 and F16 was **205.000000**.

Observation 3.2

measured by the tangent, the brighter the setting, the further from the square law. At f-number 16 the ratio is 0.999024 and the two measures are nearly the same, but at 1 it is 0.8 and at 0.5 a half: measured by the tangent, however small the f-number, the half-angle never reaches 90 degrees and illuminance never reaches π .

4. Solid Angle and Projected Solid Angle

Computation 4.1: solid angle and projected solid angle of the exit cone (half-angle by sine).

f-number	solid angle	projected solid angle	projected over solid angle
0.5	6.283185	3.141593	0.500000
1.0	0.841787	0.785398	0.933013
2.0	0.199517	0.196350	0.984123

Observation 4.2

illuminance looks not at solid angle but at projected solid angle. The cone at f-number 0.5 is a hemisphere of solid angle 2π , but light arriving obliquely strikes the image plane obliquely and is weakened, so what counts for illuminance is the projected solid angle π : the larger the f-number and the narrower the cone, the closer the two angles.

5. The Separator

Observation 5.1

the separator is whether the half-angle of the aperture is measured by its sine or by its tangent. Measured by the sine, illuminance is exactly inversely proportional to the square of the f-number, the lower limit of the f-number is 0.5, and there illuminance becomes π : measured by the tangent, the ratio between F1 and F16 becomes 205 rather than 256, departing more on the bright side. On the dark side the difference between the two measures vanishes.

Observation 5.2

what is not said. Computing aberrations, brightness at the edge of the frame, transmittance and the markings of real lenses are not treated. What is shown is that the square law in the f-number is exact when measured by the sine, and that then the f-number in air cannot go below 0.5.

6. Roll Call and Closing

1. $E = \pi L \sin^2 \theta'$ (Section 1): the squared sine.
2. ratio 2 per step of $\sqrt{2}$; **256.000000** between F1 and F16 (Computation 2.1): exact halving.
3. illuminance π at f-number 0.5 (Computation 2.1): whole hemisphere, lower limit.
4. tangent over sine **0.800000** at f-number 1 (Computation 3.1): departs on the bright side.
5. by tangent, F1 over F16 is **205.000000** (Computation 3.1): not 256.
6. projected over solid angle **0.500000, 0.933013, 0.984123** (Computation 4.1): projected solid angle counts.
7. sine, or tangent (Observation 5.1): the separator; Papers 358, 757, 918.

Closing: Brightness follows the square of the f-number when the half-angle is measured by its sine, and in air the f-number cannot go below 0.5. Measured by the sine, illuminance halved exactly per step of $\sqrt{2}$, and the ratio between F1 and F16 was 256: at f-number 0.5 the half-angle is 90 degrees and illuminance reaches the hemisphere's projected solid angle π ; a smaller f-number cannot be made in air. Measured by the tangent, illuminance at f-number 1 is 0.8 times as large, and the ratio between F1 and F16 was 205: the separator is whether the half-angle of the aperture is measured by its sine or by its tangent. Placed among the earlier papers: against the depth side of the f-number shown to be a convention in Paper 358, the exactness of its brightness side was counted; the same sine as the numerical aperture of Paper 757 was shown to set the lower limit; and projected solid angle was set beside the sharing of solid angle in Paper 918. To be honest: nothing is said about which lens is better. In which way of measuring the square law is exact, and no further.

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