

# A Fifty-Fifty in Minesweeper Is Not Set by the Visible Numbers Alone, and Tilts with the Density of Mines Left on the Board

— of two solutions, the one with fewer mines weighs more on a sparse board and the one with more mines on a dense board — the separator is whether the remaining cells and remaining mines are counted in — [Paper 924]

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September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/924>

— *From thinking to syncing.* —

## Abstract

In Minesweeper, the constraints of the visible numbers sometimes allow two solutions that look equally likely. This paper takes a case where two 1s each require one mine, among an end cell and the middle and among the middle and the other end, counts the probability that the middle is a mine with the unknown cells and mines left on the board included, and checks it against exhaustive counts on small boards. No new theorem or law is claimed. First, by the visible numbers alone it is fifty-fifty, but including the rest of the board fixes a formula: the solutions are “only the middle is a mine” (one mine) and “both ends are mines” (two); with  $U$  remaining cells and  $M$  remaining mines, the probability for the middle is set by the ratio of the number of ways to place the rest in each solution, and it matched exhaustive counts on five small boards exactly as fractions (Section 2). Second, on a sparse board, the solution with fewer mines weighs more: with  $U = 78$  and  $M = 10$  (density 0.123457) the probability that the middle is a mine is **0.886076**; with  $U = 60$ ,  $M = 9$  it is 0.868852; with  $U = 100$ ,  $M = 20$ , 0.811881 (Section 3). Third, on a dense board, the solution with more mines weighs more: with  $U = 20$ ,  $M = 12$  it is 0.476190, and with  $U = 12$ ,  $M = 11$  (density 0.733333) **0.230769**; it is exactly  $1/2$  when  $U = 2M - 3$  (Section 3). Fourth, the probability for an end cell equals that for any remaining cell on the board: in all seven cases, the probability that an end is a mine and the probability that a remaining cell is a mine had the same value (for example 0.113924) (Section 3). Fifth, the separator is whether the remaining cells and remaining mines are counted in: if not, the two solutions weigh the same and it is fifty-fifty; if so, the number of ways to place the rest becomes each solution’s weight, and the balance tilts with density (Section 4). **Keywords:** Minesweeper, conditional probability, binomial coefficient, mine density, counting configurations, constraint satisfaction.

Scope of this paper (scope note): No new theorem or law is claimed: counting configurations that satisfy constraints to obtain probabilities, counting placements by binomial coefficients, and the difficulty of deciding Minesweeper (NP-completeness) are all known. No winning strategy is discussed, where to open next, rules making the first move safe, and tables of probabilities are not treated. The case is limited to one pattern: only the constraints two 1s place on three cells, and the remaining cells untouched by them, are counted. Board sizes are set values: the pairs of  $U$  and  $M$  are chosen to vary density and claim nothing about boards of particular difficulty. Relation to earlier papers: Paper 470 showed that even a test ninety-nine percent accurate can be wrong nine times in ten. Here too, with the same visible clues, the answer tilts with a prior proportion, the fraction of mines on the whole board. Paper 611 showed that the chance of winning by switching doors is decided by how the host opens them. Here, with the visible numbers appearing the same, the probability is decided by how the unseen rest of the board is counted. What is added: writing the weights of the two solutions as a ratio of placement counts and checking it exhaustively, setting down the probability at seven densities, finding the fifty-fifty condition  $U = 2M - 3$ , showing that the end cell and the rest of the board have equal probabilities, and placing the separator on “counting the remaining cells and mines in, or not”.

## 1. Notation and the Question

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Setting: for three unknown cells in a row  $x_1, x_2, x_3$ , two visible 1s impose  $x_1 + x_2 = 1$  and  $x_2 + x_3 = 1$ . The solutions are  $\{x_2\}$  (one mine) and  $\{x_1, x_3\}$  (two). With  $U$  other unknown cells not touched by these clues and  $M$  mines left in all, taking every configuration as equally weighted:

$$P(x_2) = \frac{\binom{U}{M-1}}{\binom{U}{M-1} + \binom{U}{M-2}}, \quad \frac{\binom{U}{M-1}}{\binom{U}{M-2}} = \frac{U - M + 2}{M - 1}$$

The check compares this formula as a fraction with an exhaustive count placing  $M$  mines on  $U + 3$  cells.

Question: with the visible numbers the same, what decides the probability that the middle is a mine?

## 2. Formula and Exhaustive Count

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Computation 2.1: exhaustive counts on small boards.

| $U, M$ | formula            | exhaustive count |
|--------|--------------------|------------------|
| 6, 2   | $6/7 = 0.857143$   | 6/7              |
| 6, 4   | $4/7 = 0.571429$   | 4/7              |
| 8, 3   | $7/9 = 0.777778$   | 7/9              |
| 10, 6  | $6/11 = 0.545455$  | 6/11             |
| 12, 4  | $10/13 = 0.769231$ | 10/13            |

### Observation 2.2

by the visible numbers alone it is fifty-fifty, but including the rest of the board the probability follows the formula. The two solutions satisfy the visible numbers equally, but differ in the number of ways to place the remaining mines on the remaining cells: the one-mine solution places  $M - 1$  mines on the rest, the two-mine solution  $M - 2$ .

## 3. Varying the Density

Computation 3.1: seven cases.

| $U$ | $M$ | density $M/(U + 3)$ | $x_2$ is a mine | end $x_1$ , or a remaining cell, is a mine |
|-----|-----|---------------------|-----------------|--------------------------------------------|
| 78  | 10  | 0.123457            | 0.886076        | 0.113924                                   |
| 60  | 9   | 0.142857            | 0.868852        | 0.131148                                   |
| 100 | 20  | 0.194175            | 0.811881        | 0.188119                                   |
| 40  | 12  | 0.279070            | 0.731707        | 0.268293                                   |
| 30  | 16  | 0.484848            | 0.516129        | 0.483871                                   |
| 20  | 12  | 0.521739            | 0.476190        | 0.523810                                   |
| 12  | 11  | 0.733333            | 0.230769        | 0.769231                                   |

The probability for the middle is exactly  $1/2$  when  $U = 2M - 3$  (it was  $1/2$  at  $U = 21, M = 12$  and  $U = 9, M = 6$ ). A beginner board of  $9 \times 9$  with 10 mines has  $\binom{81}{10} = 1.8784 \times 10^{12}$  configurations.

### Observation 3.2

on a sparse board the solution with fewer mines weighs more, on a dense board the one with more. At density 0.123457 the probability that the middle is a mine is 0.886076, and at density 0.733333 it falls to 0.230769: the probability for an end cell came out equal, in every case, to that for a remaining cell on the board.

## 4. The Separator

### Observation 4.1

the separator is whether the remaining cells and remaining mines are counted in. If not, the two solutions weigh the same and the middle looks fifty-fifty: if so, the number of ways each solution leaves to place mines on the rest becomes its weight, and their ratio  $(U - M + 2)/(M - 1)$  goes above or below 1 with density.

### Observation 4.2

what is not said. Where to open, winning strategies, the computational difficulty of deciding, and probability tables for real boards are not treated. What is shown is that a fifty-fifty of the same appearance tilts with the density of the unseen rest of the board.

## 5. Roll Call and Closing

1. ratio of placement counts  $(U - M + 2)/(M - 1)$  (Section 1): the weight of a solution.
2. formula and exhaustive count agree in five cases (Computation 2.1): as fractions.
3. **0.886076** at density 0.123457 (Computation 3.1): a sparse board.
4. **0.230769** at density 0.733333 (Computation 3.1): a dense board.
5.  $1/2$  at  $U = 2M - 3$  (Computation 3.1): the fifty-fifty condition.
6. counting the rest in, or not (Observation 4.1): the separator; Papers 470 and 611.

**Closing: A fifty-fifty in Minesweeper is not set by the visible numbers alone, and tilts with the density of mines left on the board. The two solutions of the constraints set by two 1s satisfy the visible numbers equally, but differ in the number of ways to place the remaining mines on the remaining cells: on a sparse board (density 0.123457) the probability that the middle is a mine was 0.886076, on a dense board (density 0.733333) 0.230769, and exactly fifty-fifty when  $U = 2M - 3$ . The probability for an end cell came out equal to that for a remaining cell: the separator is whether the remaining cells and remaining mines are counted in. Placed among the earlier papers: the pattern of Paper 470, where a prior proportion tilts the answer, was counted on a board, and beside Paper 611, where the host's way of opening decides, a case was set where the counting of the unseen rest decides. To be honest, where to open is not said. What tilts a fifty-fifty of the same appearance, and no further.**

## References

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1. Kaye, R. (2000). Minesweeper is NP-complete. *The Mathematical Intelligencer*.
2. Yamagishi, Y. (2026). *A Test That Is Ninety-Nine Per Cent Accurate Can Be Wrong Nine Times in Ten* [Paper 470]. YOI: <https://www.yamagishiyuuki.com/yoi/470>; DOI: 10.5281/zenodo.22733434
3. Yamagishi, Y. (2026). *The Chance of Winning by Switching Doors Is Decided by How the Host Opens Them* [Paper 611]. YOI: <https://www.yamagishiyuuki.com/yoi/611>; DOI: 10.5281/zenodo.23014472

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