

At $t = 1, -1, i$ and a Cube Root of Unity, the Jones Polynomial Gives Left and Right Trefoils the Same Value

— at those points the polynomial returns to classical quantities, the value 1 shared by every knot, the determinant and the Arf invariant — the separator is whether t is placed at a root of unity of order one to four or at a general value — [Paper 928]

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— *From thinking to syncing.* —

Abstract

The Jones polynomial is known as a knot invariant that tells the trefoil from its mirror image, left from right. This paper computes the Jones polynomials of four knots and one link from descriptions of the crossings of their diagrams, checks their values at roots of unity of low order against independently computed determinants, and counts where the difference from the mirror image disappears. No new theorem or law is claimed. First, at $t = 1$ and at a cube root of unity every knot gives 1: for the trefoil, the figure-eight knot, 5_1 and 5_2 , $V(1) = 1$ (difference 0) and $V(\omega) = 1$ (difference 1.6×10^{-15}). The two-component Hopf link has $V(1) = -2$ (Section 2). Second, at $t = -1$ the polynomial returns to the determinant: $|V(-1)|$ was 3, 5, 5 and 7, agreeing for all four with the determinant computed independently from the colouring relations at the crossings (Section 2). Third, at $t = i$ the polynomial returns to the sign of the Arf invariant: $V(i)$ was $-1, -1, -1$ and $+1$, agreeing for all four with the sign set by the determinant modulo 8 (3, 5, 5, 7) (Section 3). Fourth, at those four points left and right trefoils share a value, and at general t they do not: the mirror polynomial, computed independently from the diagram with every crossing switched, agreed with $V(1/t)$ for all four knots. The difference between the trefoil and its mirror was 0 at $t = 1, \omega, -1$ and i , **6.562500** at $t = 2$ and **1.902113** at the point at angle 36 degrees on the unit circle (Section 4). Fifth, the separator is whether t is placed at a root of unity of order one to four or at a general value: at those four points the polynomial returns to classical quantities and gives the same value as the mirror image. The power to tell left from right appears only at general values of t (Section 5). **Keywords:** Jones polynomial, Kauffman bracket, trefoil knot, mirror image, knot determinant, Arf invariant.

Scope of this paper (scope note): No new theorem or law is claimed: computing the Jones polynomial by the Kauffman bracket, $V(1) = 1$ and $V(\omega) = 1$ for every knot, $|V(-1)|$ being the determinant, $V(i)$ being set by the Arf invariant, and the mirror polynomial being $V(1/t)$ are all known. It is not said to be a complete invariant: different knots with the same Jones polynomial exist. Only four knots and one link are computed. The theoretical background is not entered: derivations from quantum groups or operator algebras are not entered and only named. The naming convention for left and right is not entered, which trefoil is called right depends on a convention, so the orientation of the diagram used here is kept. Relation to earlier papers: Paper 254 showed that knots coming undone in four dimensions is not one phenomenon, and used the Alexander polynomial only for a one-way test. Here it is counted that special values of a different polynomial return to classical quantities such as the determinant. Paper 152 showed that there are groups with the same character table that are not isomorphic. Here too, for left and right trefoils, it is counted that a quantity with the power to distinguish loses it at special points. What is added: computing the polynomials of four knots and one link, checking the value at $t = -1$ against an independently computed determinant, checking the value at $t = i$ against the determinant modulo 8, computing the mirror image independently and showing that the left-right difference vanishes at four points, and placing the separator on “whether t is placed at a root of unity of low order”.

1. Notation and the Question

Setting: a knot diagram is written by the four edge labels at each crossing (PD code), and the Kauffman bracket $\langle K \rangle$ is computed as a sum over the two smoothings at each crossing. With the writhe w the sum of crossing signs, the Jones polynomial is:

$$\langle K \rangle = \sum_s A^{a(s)-b(s)} (-A^2 - A^{-2})^{|s|-1}, \quad V(t) = (-A^3)^{-w} \langle K \rangle \Big|_{A=t^{-1/4}}$$

$a(s)$ and $b(s)$ count the two kinds of smoothing chosen, and $|s|$ is the number of loops formed. The check computes the determinant independently as a minor of the matrix of colouring relations at the crossings, and computes the mirror image independently from the diagram in which each crossing is switched according to the direction of its over-strand. Let $\omega = e^{2\pi i/3}$.

Question: for what values of t does the Jones polynomial tell left and right trefoils apart?

2. $t = 1, \omega, -1$

Computation 2.1: four knots and one link (the trefoil in the orientation of the diagram used here).

knot	$V(t)$	$V(1)$	$V(\omega)$	$V(-1)$	determinant
trefoil	$t + t^3 - t^4$	1	1	-3	3
figure-eight	$t^{-2} - t^{-1} + 1 - t + t^2$	1	1	5	5
5_1	$-t^{-7} + t^{-6} - t^{-5} + t^{-4} + t^{-2}$	1	1	5	5
5_2	$-t^{-6} + t^{-5} - t^{-4} + 2t^{-3} - t^{-2} + t^{-1}$	1	1	-7	7
Hopf link	$-t^{-5/2} - t^{-1/2}$	-2	-1	-	2

For the four knots, $V(1)$ differed from 1 by 0, $V(\omega)$ from 1 by at most 1.6×10^{-15} , and $|V(-1)|$ agreed in all four cases with the independently computed determinant.

Observation 2.2

at $t = 1$ and ω every knot has the same value, and at $t = -1$ the polynomial returns to the determinant. The four polynomials have entirely different shapes, yet at these three points they tell the knots apart only by 1 and the determinant: the determinant is a quantity known long before the polynomial.

3. $t = i$

Computation 3.1: $V(i)$ and the Arf invariant set by the determinant modulo 8 (0 for remainders 1 or 7, 1 for 3 or 5).

knot	determinant mod 8	Arf invariant	$V(i)$
trefoil	3	1	-1
figure-eight	5	1	-1
5_1	5	1	-1
5_2	7	0	+1

Observation 3.2

at $t = i$ the polynomial returns to the sign of the Arf invariant. For all four, $V(i)$ was -1 to the power of the Arf invariant, agreeing with the value set by the determinant modulo 8. Here too the polynomial returns only a classical quantity readable from the determinant.

4. Left and Right

Computation 4.1: the mirror polynomial computed from the diagram with crossings switched (write 3 to -3 for the trefoil, -5 to 5 for 5_1 and 5_2 , 0 unchanged for the figure-eight) agreed with $V(1/t)$ for all four. Differences between the knot and its mirror:

knot	$t = 1$	$t = \omega$	$t = -1$	$t = i$	$t = 2$	$t = e^{i\pi/5}$
trefoil	0	0	0	0	6.562500	1.902113
figure-eight	0	0	0	0	0	0
5_1	0	0	0	0	76.289062	3.804226
5_2	0	0	0	0	34.453125	3.077684

Observation 4.2

at the four special points left and right share a value; only at general t do they differ. Since the mirror polynomial is $V(1/t)$, at points such as $1, \omega, -1$ and i , where the reciprocal is tied to the complex conjugate, a polynomial with integer coefficients returns the same value as the original: the difference between left and right trefoils first appeared as 6.562500 at $t = 2$ and 1.902113 at $t = e^{i\pi/5}$. The figure-eight knot and its mirror had the same polynomial.

5. The Separator

Observation 5.1

the separator is whether t is placed at a root of unity of order one to four or at a general value. At those four points the polynomial returns to quantities known before it, the value 1 shared by every knot, the determinant and the Arf invariant, and gives left and right the same value: the power to tell left from right appears only at other values of t . As with the character table of Paper 152, a quantity with the power to distinguish has places where it does not.

Observation 5.2

what is not said. That the Jones polynomial is a complete invariant, derivations from quantum groups or operator algebras, knot tables, and the naming convention for left and right are not treated. What is shown is that at four special points the polynomial returns to classical quantities and there does not tell left from right.

6. Roll Call and Closing

1. the Kauffman bracket and $V(t)$ (Section 1): computed from the crossings.
2. $V(1) = V(\omega) = 1$ (Computation 2.1): the same for all four knots.
3. $|V(-1)| = 3, 5, 5, 7$ (Computation 2.1): agrees with the independent determinant.
4. $V(i) = -1, -1, -1, +1$ (Computation 3.1): the sign of the Arf invariant.
5. left-right difference **6.562500, 1.902113** (Computation 4.1): only at general t .
6. special roots of unity, or general values (Observation 5.1): the separator; Papers 254 and 152.

Closing: At $t = 1, -1, i$ and a cube root of unity, the Jones polynomial gives left and right trefoils the same value. For four knots, the values at $t = 1$ and at a cube root of unity were all 1, the magnitude at $t = -1$ was the independently computed determinant, and the value at $t = i$ was the sign set by the determinant modulo 8: the mirror polynomial is $V(1/t)$, equal to the original at the four points, and the left-right difference of the trefoil first appeared as 6.562500 at $t = 2$. At those four points, the polynomial returns to quantities known before it: the separator is whether t is placed at a root of unity of order one to four or at a general value. Placed among the earlier papers: how special values return to the classical was counted for a different polynomial from the Alexander polynomial used as a one-way test in Paper 254, and the pattern of an invariant that fails to distinguish, from Paper 152, was placed on

left and right trefoils. To be honest: nothing is said about the power to classify knots. At which points the power to distinguish disappears, and no further.

References

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