

A Cycloidal Cam Has the Largest Peak Acceleration, Yet Leaves Less Vibration at Speed Because Its Acceleration Does Not Jump at the Ends

— doubling the rise time reduces the residual vibration of curves whose acceleration jumps at the ends toward a quarter, and of curves without the jump toward an eighth — the separator is whether the acceleration jumps at the ends — [Paper 929]

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— *From thinking to syncing.* —

Abstract

A cam turns rotation into the reciprocating motion of a follower, and its rise can follow constant-velocity, parabolic, simple harmonic, cycloidal, polynomial and other motion curves. This paper sets out the peak velocity, acceleration and jerk of five curves, and counts, varying the rise time, the vibration that a follower modelled as a single spring leaves after the rise. No new theorem or law is claimed. First, the cycloid has the largest peak acceleration: with rise height and time both set to 1, the peak acceleration is 4.000000 for the parabola, 4.934802 for the simple harmonic curve, 5.773503 for the 3-4-5 polynomial and **6.283185** for the cycloid (Section 2). Second, only the cycloid and the 3-4-5 polynomial have zero acceleration at the ends: the parabola jumps from rest to an acceleration of 4 at the ends, the simple harmonic curve to 4.934802. The peak jerk of the cycloid is 39.478417 and of the 3-4-5 polynomial 59.991000, both finite (Section 2). Third, residual vibration falls by quarters for curves that jump and toward eighths for curves that do not: each time the rise time measured in natural periods is doubled, the largest residual vibration falls by factors from 0.238095 to **0.249267** for the simple harmonic curve, from 0.140981 to **0.132907** for the cycloid, and to 0.512097 for constant velocity (Section 3). Fourth, the faster the cam turns, the more vibration a jumping curve leaves: with the rise time between 8 and 16 natural periods, the simple harmonic curve left 7.414 times, the parabola 9.496 times and constant velocity 70.684 times the vibration of the cycloid. The residual vibration agreed by two routes, the integral and the equation of motion (Section 3). Fifth, the separator is whether the acceleration jumps at the ends: peak acceleration indicates magnitude, but what decides the vibration left behind is the jump in acceleration, and the cycloid, without it, suits fast rises despite its larger peak (Section 4).
Keywords: cam, cycloidal motion, jerk, residual vibration, Duhamel integral, motion curve.

Scope of this paper (scope note): No new theorem or law is claimed: cam motion curves with their peak velocity, acceleration and jerk, and the free vibration left by a single-spring system after a load (the Duhamel integral) are all standard. No cam is designed: cam profile, pressure angle, contact stress and materials are not treated. The follower is one spring and one mass: damping, spring nonlinearity and jump due to clearance are not included. The limiting rates are not proved: a quarter and an eighth are set down only as where the ratio on doubling tends. Relation to earlier papers: Paper 741 showed that one of the constraints setting the length of a railway transition curve is jerk, the rate of change of acceleration, and that the constraints swap when cant is added. Here the same rate of change of acceleration is counted at the ends of a cam rise, and whether it jumps is shown to separate how residual vibration falls. What is added: setting out the peak velocity, acceleration and jerk and the end accelerations of five curves, matching the residual vibration by two routes, integral and equation of motion, finding for each curve how residual vibration falls when the rise time doubles, setting out the residual vibration relative to the cycloid, and placing the separator on “whether the acceleration jumps at the ends”.

1. Notation and the Question

Setting: a rise of height 1 is written as displacement $s(u)$ in dimensionless time $u \in [0, 1]$. The five curves are as follows (constant velocity $s = u$; parabola $2u^2$ in the first half and $1 - 2(1 - u)^2$ in the second).

$$s_{\text{SH}} = \frac{1 - \cos \pi u}{2}, \quad s_{\text{cycloid}} = u - \frac{\sin 2\pi u}{2\pi}, \quad s_{3-4-5} = 10u^3 - 15u^4 + 6u^5$$

With rise time T and natural angular frequency of the follower spring ω_n (natural period T_n), the relative displacement y obeys $\ddot{y} + \omega_n^2 y = -\ddot{s}(t)$, and the amplitude of vibration left after the rise is:

$$A = \frac{1}{\omega_n} \left| \int_0^T \ddot{s}(t) e^{i\omega_n t} dt \right|$$

The check computes this integral by Simpson's rule, obtains the same amplitude by integrating the equation of motion with Runge-Kutta, and takes the largest residual vibration within each interval of rise time T/T_n .

Question: why is the cycloid, with the largest peak acceleration, used for fast rises?

2. Five Curves

Computation 2.1: values with height and time set to 1 (constant velocity jumps in velocity at the ends, so its acceleration is impulsive).

curve	peak velocity	peak acceleration	end accelerations	peak jerk within the interval
parabola	2.000000	4.000000	4.000000, −4.000000	0.000000
simple harmonic	1.570796	4.934802	4.934802, −4.934802	15.503138
cycloid	2.000000	6.283185	0, 0	39.478417
3-4-5 polynomial	1.875000	5.773503	0, 0	59.991000

The peak accelerations matched 2π for the cycloid, $\pi^2/2$ for the simple harmonic curve, 4 for the parabola and $10/\sqrt{3}$ for the 3-4-5 polynomial within 6.4×10^{-8} .

Observation 2.2

the cycloid has the largest peak acceleration, but only the cycloid and the 3-4-5 polynomial do not jump at the ends. The parabola and the simple harmonic curve jump in acceleration at the instant the rise starts from rest: at the instant acceleration jumps, jerk is infinite.

3. Residual Vibration

Computation 3.1: matching the integral and the equation of motion. At $T/T_n = 2.3$, the residual vibration of the simple harmonic curve was 0.029156014 by the integral and 0.029156014 by the equation of motion, and of the cycloid 0.026098926 and 0.026098926.

Computation 3.2: the largest residual vibration in each interval of T/T_n (in units of the height).

curve	[2, 4]	[4, 8]	[8, 16]	[16, 32]
constant velocity	1.283376×10^{-1}	7.087302×10^{-2}	3.738970×10^{-2}	1.914715×10^{-2}
parabola	4.718618×10^{-2}	1.647056×10^{-2}	5.022999×10^{-3}	1.398004×10^{-3}
simple harmonic	6.666667×10^{-2}	1.587302×10^{-2}	3.921569×10^{-3}	9.775171×10^{-4}
cycloid	2.670426×10^{-2}	3.764799×10^{-3}	5.289665×10^{-4}	7.030317×10^{-5}
3-4-5 polynomial	4.109291×10^{-2}	5.740077×10^{-3}	8.015945×10^{-4}	1.061633×10^{-4}

The ratios on doubling the interval were 0.552239, 0.527559, 0.512097 for constant velocity; 0.349055, 0.304968, 0.278321 for the parabola; 0.238095, 0.247059, **0.249267** for the simple harmonic curve; 0.140981, 0.140503, **0.132907** for the cycloid; and 0.139685, 0.139649, 0.132440 for the 3-4-5 polynomial. In the interval [8, 16] the ratios to the cycloid were 70.684 for constant velocity, 9.496 for the parabola, 7.414 for the simple harmonic curve and 1.515 for the 3-4-5 polynomial.

Observation 3.3

how residual vibration falls divides on whether acceleration jumps at the ends. The simple harmonic curve and the parabola, which jump in acceleration, fall by quarters each doubling; the cycloid and the 3-4-5 polynomial, which do not, fall toward eighths; constant velocity, which jumps in velocity, tends to halves: if the rise is slow enough compared with the natural period every curve leaves little vibration, but the faster it is, the more a jumping curve leaves.

4. The Separator

Observation 4.1

the separator is whether the acceleration jumps at the ends. Peak acceleration is a guide to the size of the force on the follower: but what decides the vibration left after the rise is the jump in acceleration; the cycloid, without it, leaves only 1/7.414 of the simple harmonic curve's vibration for rises of 8 to 16 natural periods, despite having the largest peak acceleration.

Observation 4.2

what is not said. Cam profile, pressure angle, contact stress, damping, jump due to clearance and proofs of the limiting rates are not treated. What is shown is that the ranking by peak acceleration and the ranking by residual vibration can be opposite, and that what separates them is the jump of acceleration at the ends.

5. Roll Call and Closing

1. $A = |\int \ddot{s} e^{i\omega_n t} dt| / \omega_n$ (Section 1): residual vibration.
2. cycloid peak acceleration **6.283185** (Computation 2.1): the largest.

3. zero end acceleration only for the cycloid and 3-4-5 (Computation 2.1): **no jump**.
4. integral and equation of motion agree (Computation 3.1): **two routes**.
5. doubling: simple harmonic **0.249267**, cycloid **0.132907** (Computation 3.2): **toward a quarter and an eighth**.
6. simple harmonic leaves **7.414** times the cycloid (Computation 3.2): **at speed, jumping curves vibrate**.
7. jump at the ends, or not (Observation 4.1): **the separator; Paper 741**.

Closing: A cycloidal cam has the largest peak acceleration, yet leaves less vibration at speed because its acceleration does not jump at the ends. Of five curves, the cycloid's peak acceleration of 6.283185 was the largest, but only the cycloid and the 3-4-5 polynomial rise from zero acceleration smoothly at the ends: each time the rise time doubled, residual vibration fell by 0.249267 for the jumping simple harmonic curve and by 0.132907 for the non-jumping cycloid, and for rises of 8 to 16 natural periods the simple harmonic curve left 7.414 times the cycloid's vibration. The rankings by peak acceleration and by residual vibration can be opposite: the separator is whether the acceleration jumps at the ends. Placed among the earlier papers: the constraint on the rate of change of acceleration that Paper 741 found in transition curves was counted at the ends of a cam rise and shown as the rate at which residual vibration falls. To be honest, which curve a cam should be made with is not said. Whether the peak or the jump decides the vibration, and no further.

References

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