

With the Same Mean Residence Time, Splitting a Mixed Vessel into More Tanks in Series Lets Less Leave Early and Brings Reaction Close to Plug Flow

— adding tanks leaves the mean unchanged, but the spread of residence times shrinks in inverse proportion to the number of tanks, and what leaves early decides how much reactant remains — the separator is whether the mean residence time is looked at or its spread — [Paper 931]

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Abstract

In a vessel that is well mixed while fluid flows through, some of what enters leaves at once and some stays long. This paper keeps the total mean residence time fixed, splits the well-mixed vessel into N tanks in series, counts how the fractions leaving early and late and the remaining fraction of a first-order reactant change, and confirms them with random particles. No new theorem or law is claimed. First, with one tank, nearly four tenths leave before half the mean time: in a single tank the fraction leaving before half the mean is **0.393469**, and the fraction leaving after twice the mean is **0.135335** (Section 2). Second, adding tanks reduces both what leaves early and what leaves late: the early fraction became **0.264241** with two tanks, **0.108822** with five, **0.031828** with ten and **0.000418** with thirty, and the variance of residence time shrank to $1/N$ of the squared mean (Section 2). Third, the remaining first-order reactant approaches plug flow: with the product of rate constant and mean residence time equal to 2, the remaining fraction fell monotonically from **0.333333** with one tank to **0.250000** with two, **0.161506** with ten and **0.138033** with a hundred, approaching the **0.135335** of plug flow with infinitely many tanks (Section 3). Fourth, random particles give the same: giving 20000 particles random residence times in each tank, five tanks gave an early fraction of **0.104150** (formula **0.108822**) and a remaining fraction of **0.184687** (formula **0.185934**) (Section 4). Fifth, the separator is whether the mean residence time is looked at or its spread: the mean is the same for any number of tanks, but the remaining first-order reactant is pulled up by what slips through early, and the more the spread shrinks, the closer the outlet comes to plug flow (Section 5). **Keywords:** residence time distribution, well-mixed tank, tanks-in-series model, plug flow, first-order reaction, Erlang distribution.

Scope of this paper (scope note): No new theorem or law is claimed: the exponential distribution of residence time in a well-mixed tank, the Erlang distribution for tanks in series, the formula for remaining first-order reactant, and the comparison with plug flow are all known in reaction engineering. No values for real equipment or environments are claimed: the product 2 of rate constant and mean residence time is a set value and corresponds to no particular reactor, lake or atmosphere. Only first-order reaction is treated: second-order reactions, diffusion, backflow and exchange between tanks are not included. Design is not discussed, which number of tanks is better, and cost, are not treated. Relation to earlier papers: Paper 865, in its scope note, did not add boxes and treated its partner as a single box without treating mixing within it; this paper sits in that seat of adding boxes (no earlier occupant). Paper 549 showed that even when the time inside changes almost sixfold, the product relation among number inside, arrival rate and time inside does not move. Here the mean residence time is kept and only the way of splitting changes, counting what slips through early, which the mean does not show. Paper 576 showed that making a water main wider makes the water in it older. Here it is counted that not the average age but the spread of ages within the same average changes what comes out. What is added: setting out early and late fractions by number of tanks, showing the variance inversely proportional to the number of tanks, counting the monotone approach of remaining first-order reactant to plug flow, confirming the formulas with random particles, and placing the separator on “the mean, or the spread”.

1. Notation and the Question

Setting: let the total mean residence time be $\tau = 1$, and split a well-mixed vessel into N tanks in series, each with mean $1/N$. The total residence time t follows an Erlang distribution. The remaining fraction of a first-order reactant (rate k) is the average of e^{-kt} weighted by residence time.

$$P(t \leq x) = 1 - \sum_{j=0}^{N-1} e^{-Nx} \frac{(Nx)^j}{j!}, \quad \text{Var}(t) = \frac{1}{N}, \quad \text{remaining} = \left(1 + \frac{k\tau}{N}\right)^{-N}$$

As $N \rightarrow \infty$ the remaining fraction becomes the plug-flow value $e^{-k\tau}$. The check sets $k\tau = 2$ and gives 20000 particles residence times summed from exponential random numbers for each tank, comparing with the formulas.

Question: if the mean residence time is the same, does the way of splitting the vessel change what comes out?

2. Leaving Early and Leaving Late

Computation 2.1: fractions by number of tanks.

tanks N	before half the mean	after twice the mean	variance
1	0.393469	0.135335	1.000000
2	0.264241	0.091578	0.500000
3	0.191153	0.061969	0.333333
5	0.108822	0.029253	0.200000
10	0.031828	0.004995	0.100000
30	0.000418	0.000007	0.033333
100	0.000000	0.000000	0.010000

Observation 2.2

adding tanks keeps the mean and reduces both what leaves early and what leaves late. With one tank, 0.393469 leaves before half the mean; split into ten, it is 0.031828: the spread about the mean shrinks, the variance being $1/N$ of the squared mean.

3. Remaining First-Order Reactant

Computation 3.1: remaining fraction for $k\tau = 2$.

tanks N	remaining fraction
1	0.333333
2	0.250000
3	0.216000
5	0.185934
10	0.161506
30	0.144257
100	0.138033
plug flow	0.135335

Observation 3.2

the remaining first-order reactant decreases monotonically with more tanks and approaches plug flow. The remaining fraction is the average of e^{-kt} over residence times, and this function is large on the short-time side, so what slips through early pushes the remainder up: one tank's 0.333333 is more than twice the plug-flow 0.135335, and a hundred tanks bring it down to 0.138033.

4. Checking with Random Particles

Computation 4.1: 20000 particles.

N	early (random)	early (formula)	remaining (random)	remaining (formula)
1	0.395100	0.393469	0.334997	0.333333
5	0.104150	0.108822	0.184687	0.185934
30	0.000400	0.000418	0.144473	0.144257

The mean residence times were 0.993630, 1.001475 and 1.000035 and the variances 1.009950, 0.196102 and 0.034189 (formulas 1, 0.2 and 0.033333), all within 0.01.

5. The Separator

Observation 5.1

the separator is whether the mean residence time is looked at or its spread. By the mean alone, one tank and a hundred look like the same equipment: by the spread, one tank lets much slip through early, and the remaining first-order reactant is more than twice that of plug flow. The more tanks shrink the spread, the closer the outlet comes to plug flow.

Observation 5.2

what is not said. Values for real reactors or environments, second-order reactions, diffusion, backflow and which number of tanks is better are not treated. What is shown is that with the same mean residence time, the way of splitting changes what leaves early and how much reactant remains.

6. Roll Call and Closing

1. Erlang distribution, variance $1/N$, remaining $(1 + k\tau/N)^{-N}$ (Section 1): **split tanks**.
2. one tank: **0.393469** leaves early (Computation 2.1): **nearly four tenths**.
3. ten tanks: **0.031828** (Computation 2.1): **less leaves early**.
4. remaining from **0.333333** to **0.138033** (Computation 3.1): **toward plug flow 0.135335**.
5. random and formula within **0.01** (Computation 4.1): **confirmed with particles**.
6. the mean, or the spread (Observation 5.1): **the separator; the scope note of Paper 865; Papers 549 and 576**.

Closing: With the same mean residence time, splitting a mixed vessel into more tanks in series lets less leave early and brings reaction close to plug flow. With one tank, 0.393469 left before half the mean; split into ten, 0.031828: the remaining first-order reactant fell monotonically from 0.333333 with one tank to 0.138033 with a hundred, approaching the plug-flow 0.135335. The mean was the same for every split; what changed was the spread of residence times: the separator is whether the mean residence time is looked at or its spread. Placed among the earlier papers: the mixing that Paper 865 treated as a single box was counted as a number of tanks, and beside the mean relation of Paper 549 and the age of water in Paper 576, the spread invisible to the mean was set. To be honest, which equipment is better is not said. That the outlet changes even with the same mean, and no further.

References

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