

In a Flow-Through Culture the Washout Boundary Is Set Not by the Maximum Growth Rate but by the Incoming Feed, and Output Peaks Short of It

— the washout boundary is the specific growth rate at the incoming feed concentration, output per hour peaks at a flow a little over eight tenths of the boundary, and the closer to the boundary the longer the culture takes to settle — the separator is whether the flow is compared with the maximum specific growth rate or with the specific growth rate at the incoming feed concentration — [Paper 933]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/933>

— *From thinking to syncing.* —

Abstract

In a vessel where a nutrient solution flows in at a constant rate and out at the same rate, microbes are washed out when growth loses to the flow. This paper places microbes following the Monod equation in a flow-through vessel, derives the washout boundary, the steady cell concentration and the flow maximizing output per hour, and confirms them by time integration. No new theorem or law is claimed. First, the washout boundary is not the maximum specific growth rate: for microbes with maximum specific growth rate 0.40 per hour, half-saturation constant 0.5 g/L and yield 0.5, fed at 10.0 g/L, the boundary dilution rate was the specific growth rate at the incoming feed concentration, **0.380952** per hour (Section 2). Second, short of the boundary the cells fall sharply, and beyond it they are washed out: the steady cell concentration was 4.964286 g/L at dilution rate 0.05 per hour, 4.250000 at 0.3, 1.916667 at 0.37 and 0.250000 at 0.38, and 0 at 0.385 and 0.39 beyond the boundary (Section 2). Third, output per hour peaks at a flow a little over eight tenths of the boundary: the product of dilution rate and cell concentration reached its maximum of 1.283485 g/L per hour at **0.312713** per hour (0.820871 of the boundary), agreeing with a grid search (Section 3). Fourth, the closer to the boundary, the longer the culture takes to settle: reaching within 1 percent of the steady cell concentration took 65.6 hours at dilution rate 0.3, 597.4 hours at 0.37 and **6060.8** hours at 0.38. When first cut off at 600 hours, the results near the boundary did not match the steady state (Section 4). Fifth, the separator is whether the flow is compared with the maximum specific growth rate or with the specific growth rate at the incoming feed concentration: compared with the fastest the microbes can grow, the boundary is misread as 0.40 per hour. On the incoming feed they cannot grow that fast, so the boundary is lower, at 0.380952 per hour (Section 5). **Keywords:** continuous culture, chemostat, washout, dilution rate, Monod equation, maximum productivity.

Scope of this paper (scope note): No new theorem or law is claimed: the Monod equation, the steady state and washout boundary of continuous culture, and the formula for the dilution rate maximizing output are all known. Ecology is not discussed: red tides, the flushing of lakes or bays, and real species of algae or microbes are not treated. Only the model of a flow-through vessel is counted. The values are representative: maximum specific growth rate 0.40 per hour, half-saturation constant 0.5 g/L and yield 0.5 are the representative values of Paper 680, and the feed concentration of 10.0 g/L is a set value. Death and attachment are not included: cell death, wall attachment and competition among species are not included. No formula for settling time is claimed: the reciprocal of the distance from the boundary is set down only as a guide. Relation to earlier papers: Paper 680, in its scope note, did not enter continuous culture, did not treat chemostat theory, dilution rate or washout, and looked only at a single batch; this paper sits in that seat of continuous culture (no earlier occupant). In its body, Paper 680 showed that in a batch, half of the growth sits in the last doubling before the substrate runs out. Here the substrate keeps flowing, and instead of running out, the point where flow and growth balance is counted. What is added: deriving that the washout boundary is the specific growth rate at the incoming feed concentration, finding the steady cell concentration at nine flows both by formula and by integration, matching the output-maximizing flow by formula and grid, counting that settling takes longer near the boundary, and placing the separator on “comparing with the maximum specific growth rate, or with the rate on the incoming feed”.

1. Notation and the Question

Setting: a vessel of unit volume receives feed of concentration S_0 at dilution rate D and discharges at the same rate. Substrate S and cells X follow the Monod equation:

$$\frac{dS}{dt} = D(S_0 - S) - \frac{\mu(S)X}{Y}, \quad \frac{dX}{dt} = (\mu(S) - D)X, \quad \mu(S) = \frac{\mu_{\max}S}{K_s + S}$$

At steady state $S^* = K_s D / (\mu_{\max} - D)$ and $X^* = Y(S_0 - S^*)$, and if $D \geq \mu(S_0)$ the cells wash out. The dilution rate maximizing output DX^* is $D_{\text{opt}} = \mu_{\max} (1 - \sqrt{K_s / (K_s + S_0)})$. The check uses $\mu_{\max} = 0.40$ per hour, $K_s = 0.5$ g/L, $Y = 0.5$ and $S_0 = 10.0$ g/L, and integrates by Runge-Kutta for 20000 hours from 0.05 g/L of cells.

Question: what sets the boundary within which microbes keep growing in a flow-through vessel, and at what flow is output greatest?

2. The Washout Boundary and Steady States

Computation 2.1: the boundary dilution rate was $\mu(S_0) = 0.380952$ per hour. Steady states at nine flows:

dilution rate D (per h)	steady substrate (g/L)	steady cells (g/L)	integrated cells	output DX^*
0.050000	0.071429	4.964286	4.964286	0.248214
0.100000	0.166667	4.916667	4.916667	0.491667
0.200000	0.500000	4.750000	4.750000	0.950000
0.300000	1.500000	4.250000	4.250000	1.275000
0.312713	1.791288	4.104356	4.104356	1.283485
0.370000	6.166667	1.916667	1.916667	0.709167
0.380000	9.500000	0.250000	0.250000	0.095000
0.385000	10.000000	0	0.000000	0
0.390000	10.000000	0	0.000000	0

For the seven flows below the boundary the integrated cells matched the steady formula within 10^{-6} , and for the two above it they fell below 10^{-6} .

Observation 2.2

the washout boundary is not the maximum specific growth rate but the specific growth rate at the incoming feed concentration. To persist, cells must grow faster than they are flushed out, and even where the substrate is richest, growth rises only to $\mu(S_0)$: just short of the boundary, at 0.38 per hour, the cells had fallen to 0.058824 of their concentration at 0.3 per hour.

3. The Output Maximum

Computation 3.1: the dilution rate maximizing output DX^* is 0.312713 per hour by formula (0.820871 of the boundary), with output 1.283485 g/L per hour; a grid search gave the same output at $D = 0.3127$ per hour.

Observation 3.2

output per hour peaks at a flow a little over eight tenths of the boundary. A faster flow discharges more liquid per hour, but near the boundary the cell concentration drops sharply: the balance of the two puts the output peak well short of the boundary.

4. Time to Settle

Computation 4.1: the time after which the cells stay within 1 percent of the steady value (for washed-out flows, below 10^{-3} g/L).

dilution rate D (per h)	time entered (h)	reciprocal of distance from boundary (h)
0.05	14.6	-
0.3	65.6	-
0.37	597.4	91.3
0.38	6060.8	1050.0
0.385	956.0	247.1
0.39	430.3	110.5

When first cut off at 600 hours, the cells at 0.38 per hour were 0.077116, not matching the steady 0.25; extending the integration brought agreement.

Observation 4.2

the closer to the boundary, the longer the culture takes to settle. On both sides of the boundary, flows nearer to it took longer to reach steady state, 6060.8 hours at 0.38 per hour: viewed over a short time, a vessel near the boundary has not yet settled, and whether it will keep growing or wash out can be misread.

5. The Separator

Observation 5.1

the separator is whether the flow is compared with the maximum specific growth rate or with the specific growth rate at the incoming feed concentration. Compared with the maximum of 0.40 per hour, any slower flow seems to allow growth: on the incoming feed, growth rises only to 0.380952 per hour, and flows of 0.385 and 0.39 per hour washed the cells out. The output peak lies further back, at 0.312713 per hour.

Observation 5.2

what is not said. The ecology of red tides, lakes or bays, values for real microbes, death, attachment, competition among species, and a formula for settling time are not treated. What is shown is that the boundary for persistent growth in a flow-through vessel is set by the incoming feed concentration, that output peaks short of it, and that settling is slow near it.

6. Roll Call and Closing

1. continuous culture under the Monod equation (Section 1): **flow and growth.**
2. boundary **0.380952** per hour (Computation 2.1): **specific growth rate on the incoming feed.**
3. cells **0.250000** at **0.38** per hour (Computation 2.1): **a sharp fall short of the boundary.**
4. output maximum at **0.312713** per hour, **0.820871** of the boundary (Computation 3.1): **a little over eight tenths.**
5. **6060.8** hours to settle at **0.38** per hour (Computation 4.1): **slow near the boundary.**
6. compared with the maximum, or with the incoming feed (Observation 5.1): **the separator; the scope note of Paper 680.**

Closing: In a flow-through culture the washout boundary is set not by the maximum growth rate but by the incoming feed, and output peaks short of it. Feeding 10.0 g/L to microbes with a maximum specific growth rate of 0.40 per hour gave a washout boundary of 0.380952 per hour, and flows beyond it washed the cells out: output per hour peaked at 0.312713 per hour, 0.820871 of the boundary, and just short of the boundary, at 0.38 per hour, settling took 6060.8 hours. Compared with the maximum growth rate, the boundary is misread: the separator is whether the flow is compared with the maximum specific growth rate or with the specific growth rate at the incoming feed concentration. Placed among the earlier papers: the continuous culture that Paper 680 did not enter,

looking only at a single batch, was counted as a flow-through vessel with the same representative values. To be honest. What happens in seas or lakes is not said. Where the balance of flow and growth breaks, and no further.

References

1. Monod, J. (1950). La technique de culture continue: théorie et applications. *Annales de l'Institut Pasteur*.
2. Novick, A., & Szilard, L. (1950). Description of the chemostat. *Science*.
3. Yamagishi, Y. (2026). *What Is Frightening About Exponential Growth Is Not Its Speed but That Half of It Sits in the Last Doubling* [Paper 680]. YOI: <https://www.yamagishiyuuki.com/yoi/680>; DOI: 10.5281/zenodo.23014794

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)