

Diminishing Returns Answers What Happens When One Input Grows; Scale All Inputs Together and Output Can More Than Double

— each added worker adds less, yet increasing capital and workers in the same proportion raises output by more than that proportion — the separator is whether one input alone is increased or all inputs in the same proportion

— [Paper 934]

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— *From thinking to syncing.* —

Abstract

“Diminishing returns” refers to the addition per unit of input shrinking as the input increases. This paper sets production from two inputs, capital and workers, as a product of powers (the Cobb-Douglas form), and counts for three pairs of exponents how output grows when only workers increase and when both increase in the same proportion. No new theorem or law is claimed. First, the marginal product of workers falls as workers increase: with capital exponent 0.3 and worker exponent 0.8, holding capital at 1 and raising workers to 1, 2, 4 and 8, the addition from one more worker fell as 0.800000, 0.696440, 0.606287 and 0.527803 (Section 2). Second, doubling workers alone does not double output: doubling only workers multiplied output by **1.741101**, and going from 1 to 10 workers lowered output per worker to **0.630957** (Section 2). Third, doubling capital and workers in the same proportion more than doubles output: in the same form, doubling both multiplied output by **2.143547**, and raising both tenfold raised output per worker to **1.258925** (Section 3). Fourth, diminishing returns occur in every pair, and growth with scale is set by the sum of exponents: doubling both gave **2.000000** for the pair summing to 1 and **1.866066** for the pair summing to 0.9, and in all three pairs the marginal product of workers fell. The sum of each input times its marginal product matched output times the sum of exponents within 3.9×10^{-10} (Section 3). Fifth, the separator is whether one input alone is increased or all inputs in the same proportion: diminishing returns answers the question of increasing one input; the sum of exponents answers the question of increasing all in the same proportion. They are different questions, so output can more than double with scale even while returns diminish (Section 4). **Keywords:** diminishing returns, returns to scale, marginal product, production function, Cobb-Douglas form, Euler’s theorem.

Scope of this paper (scope note): No new theorem or law is claimed: diminishing marginal product, returns to scale, the power-product production function, and Euler’s theorem on homogeneous functions are all known. No real economy is claimed: the exponent pairs 0.3 and 0.8, 0.5 and 0.5, 0.3 and 0.6 are set for comparison and correspond to no particular industry, firm or country. Prices and costs are not treated: how much to invest and the quantity maximizing profit are not treated. Which form is real is not said: whether the sum of exponents is above or below 1 is a set value here. Relation to earlier papers: Paper 387 showed that when to leave a patch is not decided inside the patch alone but by comparison with the surrounding average. Here, in a similar setting where additions shrink, it is counted that the shrinking answers the question of moving one input only, a different question from moving all. Paper 471 showed that piling more on does not help when what is short is a ratio. Here it is counted that piling on workers alone lowers output per worker, while increasing them with the capital ratio kept raises it. What is added: setting out how the marginal product of workers falls, setting doubling workers alone beside doubling all for three pairs, showing that output per worker moves in opposite directions under the two ways of increasing, confirming the sum of exponents through Euler’s sum, and placing the separator on “one alone, or all in the same proportion”.

1. Notation and the Question

Setting: production from capital K and workers L is set as a product of powers with exponents a and b . The marginal product of workers, and the output ratios on doubling workers alone and doubling all, are:

$$F(K, L) = K^a L^b, \quad \frac{\partial F}{\partial L} = bK^a L^{b-1}, \quad \frac{F(K, 2L)}{F(K, L)} = 2^b, \quad \frac{F(2K, 2L)}{F(K, L)} = 2^{a+b}$$

If $b < 1$ the marginal product of workers falls with L , and if $a + b > 1$ doubling all more than doubles output. By Euler’s theorem, $K\partial F/\partial K + L\partial F/\partial L = (a + b)F$. The check takes derivatives by central differences.

Question: when marginal product is falling, does increasing inputs always raise output by less than the proportion?

2. Increasing One Input Alone

Computation 2.1: marginal product of workers with capital held at 1.

a	b	$L = 1$	$L = 2$	$L = 4$	$L = 8$
0.3	0.8	0.800000	0.696440	0.606287	0.527803
0.5	0.5	0.500000	0.353553	0.250000	0.176777
0.3	0.6	0.600000	0.454715	0.344610	0.261165

With $a = 0.3$, $b = 0.8$, output per worker at 1, 2, 5 and 10 workers was 1.000000, 0.870551, 0.724780 and 0.630957.

Observation 2.2

increasing workers alone lowers both the addition per worker and output per worker. In all three pairs the marginal product of workers fell with the number of workers: with capital unchanged, added workers share the same capital.

3. Increasing All Inputs in the Same Proportion

Computation 3.1: output ratios on doubling workers alone and doubling all.

a	b	$a + b$	workers alone doubled	all doubled	Euler sum over F'
0.3	0.8	1.1	1.741101	2.143547	1.100000
0.5	0.5	1.0	1.414214	2.000000	1.000000
0.3	0.6	0.9	1.515717	1.866066	0.900000

The Euler sum matched the sum of exponents within 3.9×10^{-10} . With $a = 0.3$, $b = 0.8$, output per worker when capital and workers were scaled by 1, 2, 5 and 10 was 1.000000, 1.071773, 1.174619 and 1.258925.

Observation 3.2

even while the marginal product of workers falls, increasing all in the same proportion can more than double output. With $a = 0.3$, $b = 0.8$, doubling workers alone gave 1.741101 times and doubling all 2.143547 times, and output per worker moved in opposite directions under the two ways. What decides growth on doubling all is not one exponent but the sum of exponents.

4. The Separator

Observation 4.1

the separator is whether one input alone is increased or all inputs in the same proportion. The question of increasing one input is answered by whether that input's exponent is below 1, and there diminishing returns occur: the question of increasing all in the same proportion is answered by the sum of exponents, and if the sum exceeds 1 output more than doubles. "Diminishing returns" and "returns to scale" are different questions put to the same formula.

Observation 4.2

what is not said. Exponents of real industries or countries, prices and costs, how much to invest, and which form is real are not treated. What is shown is that in the same formula increasing one input alone diminishes, while increasing all can more than double.

5. Roll Call and Closing

1. $F = K^a L^b$, 2^b , 2^{a+b} (Section 1): two questions.
2. marginal product of workers from 0.800000 to 0.527803 (Computation 2.1): diminishing.
3. workers alone doubled: 1.741101 times (Computation 3.1): short of double.
4. all doubled: 2.143547 times (Computation 3.1): more than double.
5. output per worker 0.630957 and 1.258925 (Computations 2.1, 3.1): opposite directions.
6. Euler sum equals the sum of exponents (Computation 3.1): within 3.9×10^{-10} .
7. one alone, or all in the same proportion (Observation 4.1): the separator; Papers 387 and 471.

Closing: Diminishing returns answers what happens when one input grows; scale all inputs together and output can more than double. With capital exponent 0.3 and worker exponent 0.8, the marginal product of workers fell with their number, and doubling workers alone multiplied output by 1.741101: doubling capital and workers together gave 2.143547 times, and output per worker fell when only workers were added but rose when all were. Growth on increasing all is set by the sum of exponents: the separator is whether one input alone is increased or all inputs in the same proportion. Placed among the earlier papers: beside the shrinking additions of Paper 387 and the missing ratio of Paper 471, a way of putting two questions to one formula was set. To be honest, which economy has which form is not said. That diminishing returns and returns to scale are different questions, and no further.

References

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