

# **The Proportion an Up-Down Method Tracks Is Chosen by Its Rule, and with an Asymmetric Rule the Converged Level Shifts Up in Proportion to the Step**

— a rule stepping once each way converges on the half point, while rules that step down only after two or three correct answers in a row track points near seven and eight tenths — the separator is the number of consecutive correct answers required before stepping down — [Paper 935]

---

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

September 2026

YOI: <https://www.yamagishiyuuki.com/yoi/935>

— *From thinking to syncing.* —

## Abstract

The up-down method for measuring sensory thresholds weakens the stimulus after a correct answer and strengthens it after an error, gathering the stimulus around some level. This paper sets up a random observer whose proportion of correct answers follows the cumulative normal distribution, and counts, for three rules on how many consecutive correct answers trigger a step down, where the stimulus converges and what proportion of correct answers holds there. No new theorem or law is claimed. First, the rule chooses the proportion tracked: a rule stepping down after  $k$  consecutive correct answers and up after one error tracks the point where the proportion correct is the  $k$ -th root of 0.5. For  $k = 1, 2, 3$  the proportions are 0.500000, **0.707107** and 0.793701, at stimulus levels 0, 0.544952 and 0.819329 (Section 2). Second, the one-each-way rule converges exactly on its target: averaging the latter part of 1000 runs of 600 trials at step 0.1, the converged level was 0.001071 (standard error 0.001959), indistinguishable from the target 0 (Section 2). Third, with asymmetric rules the converged level shifts above the target: the two-in-a-row rule gave **0.558204** (shift +0.013252) and the three-in-a-row rule 0.836896 (+0.017567), both beyond three standard errors. Even so, the proportions correct over the converged stretch were 0.706395 and 0.792878, within 0.002 of the target proportions (Section 3). Fourth, the shift grows in proportion to the step: with the two-in-a-row rule at steps 0.1, 0.5 and 1.0, the shift was 0.012337, 0.068678 and **0.135898**, and dividing by the step gave 0.123374, 0.137356 and 0.135898 (Section 3). Fifth, the separator is the number of consecutive correct answers required before stepping down: that number decides which proportion of the observer's responses is called the threshold. When it exceeds one, steps up and down are no longer symmetric, and the stimulus level drifts upward in proportion to the step (Section 4). **Keywords:** up-down method, psychometric function, threshold, transformed up-down method, step size, estimation bias.

Scope of this paper (scope note): No new theorem or law is claimed: the up-down method, the transformed up-down method that steps down only after consecutive correct answers (Levitt), its tracking of the  $k$ -th root of 0.5, and the psychometric function are all known. Real perception is not treated: the observer is a model whose proportion correct at stimulus strength  $x$  is the standard normal cumulative  $\Phi(x)$ ; guessing in forced-choice tasks, fatigue and learning are not included. Which rule is better is not said: efficiency of estimation and the choice of number of trials are not treated. The shift coefficient is not made a theory: the shift divided by the step is set down as measured, not claimed as a formula. Relation to earlier papers: Paper 837, in its scope note, did not discuss measurement and did not enter psychophysical procedures such as the up-down and constant-stimulus methods; this paper sits in that seat of the up-down method (no earlier occupant). In its body, Paper 837 showed that the threshold of two-point discrimination has two limits, the receptive field and the receptor spacing. Here the side measuring the threshold is taken, and it is counted that for the same observer the level called threshold changes with the rule. What is added: setting out the proportions and stimulus levels tracked by three rules, obtaining converged levels and standard errors with a random observer, showing the shift of asymmetric rules while their proportions still match the targets, measuring that the shift grows in proportion to the step, and placing the separator on “the number of correct answers required before stepping down”.

## 1. Notation and the Question

---

Setting: let the proportion of correct answers at stimulus strength  $x$  be  $P(x) = \Phi(x)$ , the standard normal cumulative. After  $k$  consecutive correct answers  $x$  steps down by the step size, and after any error it steps up. The stimulus balances where stepping up and stepping down are equally likely:

$$P(x^*)^k = \frac{1}{2}, \quad x^* = \Phi^{-1}\left(2^{-1/k}\right)$$

The check starts at  $x = 2$ , repeats 600 trials 1000 times, and takes the mean  $x$  over the latter 400 trials as that run's converged level.

Question: is the level at which the up-down method converges set by the observer alone?

## 2. The Proportion Chosen by the Rule

---

Computation 2.1: three rules at step 0.1.

| rule         | target proportion | target level | converged level (s.e.) | shift     | proportion correct |
|--------------|-------------------|--------------|------------------------|-----------|--------------------|
| down after 1 | 0.500000          | 0            | 0.001071 (0.001959)    | +0.001071 | 0.500030           |
| down after 2 | 0.707107          | 0.544952     | 0.558204 (0.002077)    | +0.013252 | 0.706395           |
| down after 3 | 0.793701          | 0.819329     | 0.836896 (0.002334)    | +0.017567 | 0.792878           |

### Observation 2.2

even for the same observer, the rule chooses the level called threshold. The one-each-way rule heads for the 0.5 point, the two-in-a-row rule for 0.707107 and the three-in-a-row rule for 0.793701: the observer stays the same; only the rule for stepping down changed.

## 3. The Shift of Asymmetric Rules

Computation 3.1: the one-each-way shift of 0.001071 was within three standard errors of 0.001959, while the two- and three-in-a-row shifts exceeded three standard errors. The differences between proportion correct and target proportion were 0.000030,  $-0.000712$  and  $-0.000823$ .

Computation 3.2: varying the step for the two-in-a-row rule.

| step | converged level | shift     | shift over step |
|------|-----------------|-----------|-----------------|
| 0.1  | 0.557290        | +0.012337 | 0.123374        |
| 0.5  | 0.613630        | +0.068678 | 0.137356        |
| 1.0  | 0.680850        | +0.135898 | 0.135898        |

### Observation 3.3

with an asymmetric rule, the converged level shifts up in proportion to the step. The shift grew with the step, and divided by the step lay between 0.12 and 0.14 in every case: even so, the proportion correct nearly matched the target. The mean stimulus level and the mean proportion at those levels are quantities that need not agree.

## 4. The Separator

---

### Observation 4.1

the separator is the number of consecutive correct answers required before stepping down. When that number is one, steps up and down are symmetric, and the stimulus converges exactly on the 0.5 point: when it is two or three, stepping down requires consecutive correct answers, the symmetry of steps breaks, and the stimulus level drifts upward in proportion to the step. The threshold is not a quantity of the observer alone; it includes the conventions of rule and step.

### Observation 4.2

what is not said. Real perception, tasks with guessing, efficiency of estimation, which rule should be used, and a formula for the shift coefficient are not treated. What is shown is that the rule chooses the proportion an up-down method tracks, and that with asymmetric rules the converged level shifts in proportion to the step.

## 5. Roll Call and Closing

---

1.  $P(x^*)^k = 1/2$  (Section 1): the proportion chosen by the rule.
2. 0.500000, **0.707107**, **0.793701** (Computation 2.1): targets of three rules.
3. one-each-way rule **0.001071** (Computation 2.1): converges on target.
4. two-in-a-row rule **0.558204** (Computation 2.1): shifts up.
5. shift **0.135898** at step 1.0 (Computation 3.2): proportional to the step.
6. the number required before stepping down (Observation 4.1): the separator; the scope note of Paper 837.

**Closing:** The proportion an up-down method tracks is chosen by its rule, and with an asymmetric rule the converged level shifts up in proportion to the step. Applying rules that step down after one, two or three consecutive correct answers to the same observer, the tracked proportions were 0.500000, 0.707107 and 0.793701: the one-each-way rule converged on its target, but the two-in-a-row rule reached 0.558204, 0.013252 above target, and at step 1.0 the shift became 0.135898. Even so, the proportion correct over the converged stretch matched the target within 0.002: the separator is the number of consecutive correct answers required before stepping down. Placed among the earlier papers: the up-down method that Paper 837 did not enter was counted as a convention on the measuring side. To be honest, which rule is better is not said. That a threshold contains the conventions of rule and step, and no further.

# References

---

- 1. Dixon, W. J., & Mood, A. M. (1948). A method for obtaining and analyzing sensitivity data. *Journal of the American Statistical Association*.
- 2. Levitt, H. (1971). Transformed up-down methods in psychoacoustics. *The Journal of the Acoustical Society of America*.
- 3. Yamagishi, Y. (2026). *Two-Point Discrimination Has Two Limits: Narrowing the Receptive Field Never Beats the Receptor Spacing* [Paper 837]. YOI: <https://www.yamagishiyuuki.com/yoi/837>; DOI: 10.5281/zenodo.23015616

---

— *From thinking to syncing.* —

**Yuuki YAMAGISHI (山岸 勇輝)**

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)