

Pumping Up to the Recharge Stops Groundwater Decline, but It Stops Only When Nearly All the Water Once Flowing to the River Has Been Taken

— pumped water comes first from storage and later from discharge to the river, and at the new balance the discharge has fallen by the amount pumped — the separator is whether the allowable pumping is measured by recharge or by the discharge it captures — [Paper 937]

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Abstract

It is sometimes said that groundwater is sustainable if pumping does not exceed what rain returns (recharge). This paper sets up a single-box aquifer that receives recharge and discharges to a river, and counts, after pumping begins, where the pumped water comes from and what happens to discharge at the new balance. No new theorem or law is claimed. First, pumped water comes first from storage and later from discharge to the river: in an aquifer with a response time of 10 years, the fraction of pumped water coming from the river was **0.393424** at 5 years, **0.632102** at 10, **0.950215** at 30 and **0.997522** at 60 (formula **0.997521**) (Section 2). Second, pumping as much as the recharge nearly eliminates discharge to the river: with recharge 1 and pumping 1, discharge fell to **0.606515** at 5 years, **0.367861** at 10, **0.049780** at 30 and **0.002478** at 60. Of 60 pumped over 60 years, **9.975222** came from storage and **50.025778** from the river (Section 2). Third, at three tenths of the recharge, discharge settles at seven tenths of natural: pumping 0.3, discharge became **0.700743** at 60 years. Under the view keeping river flow at seven tenths of natural, the allowable pumping is **0.300000** of the recharge, differing by a factor of **3.333333** from the view allowing pumping up to the recharge (Section 3). Fourth, approaching the new balance takes about three response times: the time to come 95 percent of the way to balance is **29.957323** years, and the integrated discharge matched the formula within 10^{-3} (Section 3). Fifth, the separator is whether the allowable pumping is measured by recharge or by the discharge it captures: measured by recharge, pumping up to the recharge looks sustainable. Measured by captured discharge, that pumping takes all the water that flowed to the river (Section 4). Keywords: groundwater budget, recharge, pumping, stream discharge, sustainable yield, aquifer.

Scope of this paper (scope note): No new theorem or law is claimed: the groundwater budget, the supply of pumping by storage loss and reduced discharge (capture), and the critique of the view that pumping up to recharge is sustainable are all known in hydrology. No real aquifer is claimed: recharge 1 and a response time of 10 years are set values and correspond to no particular aquifer or river. Space is not included: the aquifer is a single box; the distance between well and river and the spread of drawdown are not included. How much to pump is not said: keeping river flow at seven tenths of natural is one convention set for comparison. Relation to earlier papers: Paper 599, in its scope note, did not include the return of groundwater by rain, the edges of the aquifer, or losses in the well itself; this paper sits in that seat of the return by rain (no earlier occupant). Paper 574 showed that lowering the water table by 1 m raises the bottom of the fresh water by 40 m. Here it is not the water level but, over time, whether pumped water comes from storage or from the river that is counted. What is added: setting out the source of pumped water over time, finding how discharge falls when pumping equals recharge, finding where discharge settles at three tenths of the recharge, showing the ratio of amounts allowed by the two views, and placing the separator on “measured by recharge, or by captured discharge”.

1. Notation and the Question

Setting: take the aquifer as a single box with storage V , recharge R , discharge to the river V/τ and pumping P . Before pumping, $V = R\tau$ and discharge balances recharge.

$$\frac{dV}{dt} = R - \frac{V}{\tau} - P, \quad \text{discharge}(t) = (R - P) + P e^{-t/\tau}, \quad \text{fraction from the river} = 1 - e^{-t/\tau}$$

Of the pumped water, the part equal to the reduction in discharge comes from the river and the rest from loss of storage. The check sets $R = 1$ and $\tau = 10$ years, and integrates for 60 years in steps of 0.001 year.

Question: is pumping up to the recharge sustainable, and where does the pumped water come from?

2. Pumping as Much as the Recharge

Computation 2.1: $P = 1$ (equal to the recharge).

time (years)	discharge to river	fraction of pumping from river	storage
5	0.606515	0.393424	6.065155
10	0.367861	0.632102	3.678610
30	0.049780	0.950215	0.497796
60	0.002478	0.997522	0.024780

Of 60 pumped over 60 years, 9.975222 came from storage and 50.025778 from the river. The formula gives discharge 0.002479 and a river fraction of 0.997521 at 60 years.

Observation 2.2

pumping as much as the recharge stops the decline, but nearly eliminates discharge to the river. At first storage falls, but as the water level drops less flows to the river, and that reduction comes to supply the pumping: at the new balance, the pumped water comes not from recharge but from water that would have flowed to the river.

3. Pumping Three Tenths of the Recharge

Computation 3.1: $P = 0.3$.

time (years)	discharge to river	fraction of pumping from river	storage
5	0.881955	0.393424	8.819546
10	0.810358	0.632102	8.103583
30	0.714934	0.950215	7.149339
60	0.700743	0.997522	7.007434

The formula gives discharge 0.700744 at 60 years, within 10^{-3} of the integration. The time to come 95 percent of the way to balance is $\tau \ln 20 = 29.957323$ years.

Observation 3.2

whatever the pumping rate, the fraction of pumped water from the river approaches one at the same pace. At both $P = 0.3$ and $P = 1$, the river fraction at 60 years was 0.997522. What differs is where discharge settles: seven tenths of natural for $P = 0.3$, nearly zero for $P = 1$. The amount allowed by keeping river flow at seven tenths of natural is $1/3.333333$ of that allowed by pumping up to the recharge.

4. The Separator

Observation 4.1

the separator is whether the allowable pumping is measured by recharge or by the discharge it captures. Measured by recharge, pumping up to the recharge stops the decline of groundwater and so looks sustainable: measured by captured discharge, that pumping takes nearly all the water flowing to the river, and at 60 years discharge is 0.002478 of the recharge. The amounts allowed by the two views differ by a factor of 3.333333.

Observation 4.2

what is not said. Values for real aquifers or rivers, the distance between well and river, the spread of drawdown, and how much to pump are not treated. What is shown is that the reason the decline stops under pumping up to the recharge is the exhaustion of discharge to the river, and that in the long run pumped water comes from discharge.

5. Roll Call and Closing

1. $dV/dt = R - V/\tau - P$ (Section 1): the budget of one box.
2. river fraction **0.997522** at 60 years (Computation 2.1): the source is discharge.
3. pumping equal to recharge: discharge **0.002478** (Computation 2.1): nearly gone.
4. **0.9975222** from storage, **50.025778** from river (Computation 2.1): the breakdown of 60 pumped.
5. three tenths: discharge **0.700743** (Computation 3.1): seven tenths of natural.
6. ratio of allowed amounts **3.333333** (Observation 3.2): two views.

7. recharge, or captured discharge (Observation 4.1): the separator; the scope note of Paper 599; Paper 574.

Closing: Pumping up to the recharge stops groundwater decline, but it stops only when nearly all the water once flowing to the river has been taken. Pumped water came first from storage and later from discharge to the river, and at 60 years the fraction from the river was 0.997522: pumping as much as the recharge lowered discharge to the river to 0.002478, and pumping three tenths of it let discharge settle at seven tenths of natural. In the long run, pumped water comes not from recharge but from discharge: the separator is whether the allowable pumping is measured by recharge or by the discharge it captures. Placed among the earlier papers: the return by rain left out by Paper 599 was put into the budget, and not the water-level side of Paper 574 but the source of pumped water was counted over time. To be honest: how much to pump is not said. What is used up when pumping reaches the recharge, and no further.

References

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