

The Formula for the Gain in One Split Does Not Ask Whether the Falling Particle Can Get There

— splitting near the horizon of a rotating black hole approaches the formula's maximum, but restricted to splits that a particle falling from rest far away can reach, the gain is smaller unless the spin is extreme —the separator is whether a turning point lies outside the splitting point — [Paper 939]

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— *From thinking to syncing.* —

Abstract

If a particle is split in two inside the ergosphere of a rotating black hole and one piece falls in with negative energy, the other comes out with more energy than the particle that fell in. This paper takes a particle falling from rest far away in the equatorial plane and splitting into two photons at its turning point, and numerically compares, for five spins, the largest gain allowed by conservation and the photon condition alone with the largest gain among splits the particle can actually reach. No new theorem or law is claimed. First, with local conditions alone, the largest gain matches the formula set by the horizon radius: splitting just outside the horizon (one millionth outside) for spins $a = 0.5, 0.9, 0.99$ and 0.999 , the largest gain fraction was $0.017638, 0.090098, 0.161956$ and 0.191810 , matching the formula, and for $a = 1$ it was 0.207012 , within one thousandth of the formula's 0.207107 (Section 2). Second, the particle that maximum needs cannot arrive from far away unless the spin is extreme: a particle with the angular momentum that split requires had a turning point outside the splitting point for the four spins with $a < 1$. Only $a = 1$ arrived (Section 2). Third, restricted to particles that arrive, the gain is below the formula: searching over radius and keeping only reachable splits, the largest gain was 0.037231 for $a = 0.9$ (**0.413223** of the formula), 0.142926 for 0.99 (0.882498), 0.185693 for 0.999 (0.968106) and 0.207109 for $a = 1$ (1.000012). For $a = 0.5$ this search found none (Section 3). Fourth, the farther from the horizon the split, the smaller the gain, and outside the ergosphere there is none: for $a = 1$, splitting at radii $1.01, 1.1, 1.3, 1.6$ and 1.9 gave gains of $0.203598, 0.174200, 0.120174, 0.059017$ and 0.012989 , and at 2.1 , outside the ergosphere, -0.012050 (Section 4). Fifth, the separator is whether a turning point lies outside the splitting point: the formula looks only at conservation and the photon condition at the spot. Whether a particle falling from far away is pushed back on the way lies outside the formula (Section 5). **Keywords:** Penrose process, Kerr black hole, ergosphere, turning point, energy extraction, general relativity.

Scope of this paper (scope note): No new theorem or law is claimed: the Penrose process, the ergosphere of a Kerr black hole, the formula that sets the upper limit of the gain in one split by the horizon radius, and the maximum of about 0.207107 for an extremal hole are all known in general relativity. Units are aligned: the gravitational constant, the speed of light and the hole's mass are set to 1. The form of the split is restricted: only a particle falling from rest far away in the equatorial plane and splitting into two photons at its turning point is treated. Particles thrown in, massive fragments, orbits off the equatorial plane and multi-stage splits are not treated. No real object is treated: the role as a source of jets or light from accretion discs is not discussed. Relation to earlier papers: Paper 887, in its scope note, did not enter the efficiency of a single split and did not treat the upper limit of efficiency when one particle is split in two to extract energy; this paper sits in that seat (no earlier occupant). Paper 887 showed that the total energy extractable from a rotating black hole is at most 0.292893 of the hole's mass. Here the gain in one split is counted relative to the energy of the particle dropped in. The denominators differ, so the two numbers are not compared. What is added: checking the local-only maximum against the formula for five spins, checking whether the particle of that split can arrive from far away, finding the maximum among reachable splits and its ratio to the formula, listing the gain against splitting radius and outside the ergosphere, and placing the separator on "whether a turning point lies outside the splitting point".

1. Notation and the Question

Setting: set the gravitational constant, the speed of light and the hole's mass to 1, the spin to a and the horizon radius to $r_+ = 1 + \sqrt{1 - a^2}$. In the equatorial plane, a particle falling from rest far away (energy $E_0 = 1$, angular momentum L_0) splits at its turning point r into two photons; one (E_1) falls in and the other (E_2) leaves.

$$E_1 + E_2 = 1, \quad L_1 + L_2 = L_0, \quad g^{\mu\nu} p_\mu p_\nu = 0 \text{ (photons)}, \quad \eta = E_2 - 1$$

L_0 is chosen so the particle's radial momentum is 0 at the splitting point, and both photons are future-directed (positive energy as seen by a locally rotating observer). The largest gain set by local conditions alone is known to be the following in terms of the horizon radius.

$$\eta_{\max} = \frac{1}{2} \left(\sqrt{\frac{2}{r_+}} - 1 \right)$$

Search: reachable splits were searched over 400 radii from the horizon $+10^{-6}$ to 2.0 (packed quadratically).

Whether a split is reachable was checked at 4000 points outside the splitting point up to 1000, asking whether the squared radial momentum ever goes negative (whether there is a turning point).

Question: is the formula's maximum actually obtained by a particle falling in from far away?

2. The Maximum from Local Conditions Alone

Computation 2.1: splitting 10^{-6} outside the horizon.

spin a	horizon r_+	formula maximum	numerical maximum	reachable from far away
0.5	1.866025	0.017638	0.017638	no
0.9	1.435890	0.090098	0.090098	no
0.99	1.141067	0.161956	0.161956	no
0.999	1.044710	0.191810	0.191810	no
1	1.000000	0.207107	0.207012	yes

Observation 2.2

with local conditions alone the formula’s maximum is reached, but the particle that split needs cannot arrive from far away unless the spin is extreme. For five spins the numerical maximum matched the formula within 10^{-3} : but a particle with that L_0 has a turning point outside the splitting point for the four spins with $a < 1$ and is pushed back on the way. For $a = 1$ the outgoing photon had $E_2 = 1.207012$, the infalling photon $E_1 = -0.207012$ and the particle’s angular momentum $L_0 = 2.000001$.

3. The Maximum Restricted to Particles That Arrive

Computation 3.1: the maximum over radius, keeping only reachable splits.

spin a	maximum over reachable splits	radius	formula maximum	ratio to formula
0.5	none found		0.017638	
0.9	0.037231	1.732401	0.090098	0.413223
0.99	0.142926	1.209616	0.161956	0.882498
0.999	0.185693	1.063435	0.191810	0.968106
1	0.207109	1.000007	0.207107	1.000012

Observation 3.2

restricted to particles that arrive, the gain is below the formula and approaches it as the spin nears the extreme. For $a = 0.9$ it was 0.413223 of the formula, for 0.99 and 0.999 0.882498 and 0.968106, and for $a = 1$ it matched the formula within 10^{-3} : the maximum over reachable splits does not sit at the horizon; for $a = 0.9$ it came at radius 1.732401. For $a = 0.5$ this search (splitting at the turning point, 400 radii up to 2.0) found no reachable split. It is not claimed that none exists.

4. Splitting Radius and Outside the Ergosphere

Computation 4.1: the largest gain for $a = 1$ at different splitting radii (local conditions only).

radius r	1.01	1.1	1.3	1.6	1.9
largest gain	0.203598	0.174200	0.120174	0.059017	0.012989

Computation 4.2: in the equatorial plane g_{tt} is 0.000500, 0 and -0.000500 at $r = 1.999$, 2 and 2.001, so $r = 2$ is the outer edge of the ergosphere. Outside it at $r = 2.1$, the largest gain was -0.012050 for both $a = 0.9$ and $a = 1$.

Observation 4.3

the farther from the horizon the split, the smaller the gain, and outside the ergosphere there is none. That a negative-energy photon can only be made inside the ergosphere is known, and numerically too the gain fell toward the outer edge and was negative outside: right next to the horizon for $a = 1$ the numbers are sensitive: 0.207012 at $r = 1.000001$, and 0.207109 at $r = 1.000007$ in the search over reachable splits. Here agreement with the formula is claimed only to within 10^{-3} .

5. The Separator

Observation 5.1

the separator is whether a turning point lies outside the splitting point. The formula looks only at conservation, the photon condition and future-directedness at the splitting point: whether a particle with the same L_0 can come there from far away is known only by looking at its radial momentum at every radius outside. For $a = 0.9$, looking made the gain 0.413223 of the formula.

Observation 5.2

what is not said. Particles thrown in, massive fragments, orbits off the equatorial plane, multi-stage splits and real objects are not treated. What is shown is that, for a particle falling from rest far away and splitting at its turning point, the local formula's maximum was reached only for the extremal hole.

6. Roll Call and Closing

1. conservation, photon condition, gain fraction (Section 1): local conditions.
2. numerical maximum matches the formula for five spins (Computation 2.1): reached with local conditions alone.
3. not reachable from far away for $a < 1$ (Observation 2.2): a turning point outside.
4. 0.413223 of the formula for $a = 0.9$ (Computation 3.1): maximum restricted to particles that arrive.
5. falls with splitting radius, -0.012050 at $r = 2.1$ (Computations 4.1 and 4.2): no gain outside the ergosphere.
6. a turning point outside, or not (Observation 5.1): the separator; the scope note of Paper 887.

Closing: The formula for the gain in one split does not ask whether the falling particle can get there. With conservation and the photon condition at the spot alone, the largest gain for five spins matched the formula set by the horizon radius: but the particle that split needs could not arrive from far away for the four non-extremal spins, and restricted to reachable splits the gain was 0.413223 of the formula for $a = 0.9$, 0.882498 for 0.99 and 0.968106 for 0.999. The separator is whether a turning point lies outside the splitting point. Placed among the earlier papers: the gain in one split, whose denominator differs from the total extractable energy counted in Paper 887, was counted separately according to whether the split can be reached. To be honest: particles thrown in or multi-stage splits could change the answer. How far the formula and the reachable splits lie apart for a particle falling from rest and splitting at its turning point, and no further.

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