

The Brightness Ratio of Two Colors Levels Off for Hotter Stars, and Its Power to Read Temperature Fades

— the value of the ratio changes by a constant factor between counting per wavelength and per frequency, but its sensitivity to temperature does not —the separator is whether the energy of one photon is above or below the thermal scale — [Paper 943]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

Abstract

A star's color can be expressed as the ratio of its brightness at two wavelengths, such as blue and green, and for a black body the surface temperature can be read from that ratio. This paper computes the ratio of black-body brightness at 440 nm and 550 nm for each temperature, and counts how strongly the ratio responds to temperature, how that response falls for hot stars, and what changes and what does not when the ratio is counted differently. No new theorem or law is claimed. First, the ratio rises with temperature and approaches a fixed value: the per-wavelength ratio was 0.344937 at 3000 K, 0.975636 at 5772 K, 1.528934 at 10000 K and 2.227526 at 40000 K, already **0.912395** of the high-temperature limit 2.441406 (the fourth power of the ratio of 550 to 440) (Section 2). Second, the hotter the star, the less the ratio responds to temperature: the sensitivity, the derivative of the log of the ratio with respect to the log of temperature, was 2.178742 at 3000 K, 1.103448 at 5772 K and **0.101438** at 40000 K. An error of 1 percent in the ratio becomes, to first order, 0.458980 percent in temperature at 3000 K and **9.858223** percent at 40000 K (Section 2). Third, the product of sensitivity and temperature moves from its low-temperature value to half of it: multiplying the sensitivity by temperature gives a low-temperature limit of 6539.895 K and a high-temperature limit of exactly half, 3269.947 K; at 40000 K it was 4057.526 K (Section 3). Fourth, counting differently changes the value of the ratio but not its sensitivity: the ratio counted per frequency was, at every temperature, the per-wavelength ratio divided by 1.562500, with the same sensitivity. The temperature at which the two wavelengths are equally bright differed: **5904.132** K per wavelength and **10393.120** K per frequency (Section 4). Fifth, the separator is whether the energy of one photon is above or below the thermal scale: the energy of one photon divided by the thermal scale was 10.899825 at 440 nm and 3000 K, and 0.817487 at 40000 K. On the high side the ratio responds to temperature exponentially; on the low side it settles at the fourth power of the wavelength ratio (Section 5). **Keywords:** color temperature, black-body radiation, Planck's law, surface temperature, Rayleigh-Jeans approximation, temperature sensitivity.

Scope of this paper (scope note): No new theorem or law is claimed: Planck's law, Wien's displacement law, the Rayleigh-Jeans approximation on the long-wavelength side, reading temperature from the brightness ratio at two wavelengths (color temperature), and the fact that the colors of hotter stars respond more weakly to temperature are all known. Only an ideal black body and two wavelengths are treated: the ratio of brightness at the two points 440 nm and 550 nm; actual filter widths, zero points and color index values are not entered. Observation is not treated: extinction by the atmosphere and interstellar dust, departures of starlight from a black body, and instrument scatter are not treated. "An error of 1 percent in the ratio" is a width set only to restate the sensitivity. The diagram of brightness against temperature is not entered: diagrams placing stars by brightness and temperature, and stellar evolution, are not treated. Relation to earlier papers: Paper 892, in its scope note, did not treat color and did not treat the correspondence between a star's color and surface temperature or the diagram of brightness against temperature; this paper sits in the seat of the correspondence between color and surface temperature (no earlier occupant). Paper 222 showed that the peak of sunlight appears in different places depending on whether it is counted per wavelength or per frequency. Here it is counted that for the brightness ratio at two wavelengths too, the value of the ratio and the temperature of equal brightness change with the way of counting, while the sensitivity to temperature does not. Paper 788 showed that the wavelength dividing incoming from outgoing light does not depend on the choice of variable. There, two bodies were compared at the same wavelength. Here one body is compared at two wavelengths, so the value of the ratio moves by a constant factor. What is added: the ratio and sensitivity at seven temperatures, the low- and high-temperature limits of the product of sensitivity and temperature, showing that the value of the ratio and the temperature of equal brightness change with the way of counting while the sensitivity does not, a comparison with a pair of wavelengths set further apart, and placing the separator on "whether the energy of one photon is above or below the thermal scale".

1. Notation and the Question

Setting: the per-wavelength brightness B_λ and per-frequency brightness B_ν of a black body at temperature T have, up to overall constants, the forms below ($x = hc/(\lambda kT)$ is the energy of one photon divided by the thermal scale kT).

$$B_\lambda \propto \frac{\lambda^{-5}}{e^x - 1}, \quad B_\nu \propto \frac{\lambda^{-3}}{e^x - 1}, \quad R(T) = \frac{B_\lambda(440 \text{ nm})}{B_\lambda(550 \text{ nm})}$$

The temperature read from the ratio is the color temperature. With the sensitivity to temperature $S = d \ln R / d \ln T$,

$$S = g(x_{440}) - g(x_{550}), \quad g(x) = \frac{x e^x}{e^x - 1}$$

An error of 1 percent in the ratio becomes, to first order, $1/S$ percent in temperature. The constants used are $h = 6.62607015 \times 10^{-34}$ J s, $c = 299792458$ m/s, $k = 1.380649 \times 10^{-23}$ J/K and $hc/k = 1.43877688 \times 10^{-2}$ m K. The sensitivity was found both from the formula and by a difference, moving temperature up and down.

Question: how well can the brightness ratio at two wavelengths tell apart the temperatures of hot stars, and does the answer depend on how the ratio is counted?

2. Ratio and Sensitivity

Computation 2.1: the ratio of 440 nm to 550 nm (per wavelength) and its sensitivity.

temperature K	ratio	fraction of limit	sensitivity S	temperature error for 1 percent (percent)	x_{440}	x_{550}
3000	0.344937	0.141286	2.178742	0.458980	10.899825	8.719860
4000	0.594271	0.243413	1.627815	0.614320	8.174869	6.539895
5772	0.975636	0.399621	1.103448	0.906250	5.665190	4.532152
6000	1.017328	0.416698	1.057062	0.946018	5.449912	4.359930
10000	1.528934	0.626251	0.576885	1.733448	3.269947	2.615958
20000	1.994456	0.816929	0.238265	4.197000	1.634974	1.307979
40000	2.227526	0.912395	0.101438	9.858223	0.817487	0.653989

The formula and the difference for the sensitivity agreed within 10^{-6} at all seven temperatures. The high-temperature limit of the ratio is $(550/440)^4 = 2.441406$.

Observation 2.2

the ratio rises with temperature toward a fixed value, and the sensitivity falls. At 5772 K, the value Paper 222 used for the Sun, the ratio is 0.975636 and the sensitivity 1.103448, so a 1 percent error in the ratio is a 0.906250 percent error in temperature: at 40000 K the ratio is already 0.912395 of the limit and the sensitivity 0.101438, so a 1 percent error in the ratio becomes 9.858223 percent in temperature. With the same measurement, the hotter the star, the harder its temperature is to tell apart.

3. Limits of the Product of Sensitivity and Temperature

Computation 3.1: values of $T \times S$.

temperature	$T \times S$ (K)
low-temperature limit $(hc/k)(1/440\text{nm} - 1/550\text{nm})$	6539.895
300 K	6539.895
3000 K	6536.226
5772 K	6369.104
10000 K	5768.849
40000 K	4057.526
10^8 K	3270.268
high-temperature limit (half)	3269.947

At 10^9 K the ratio was 2.441398, a relative difference of 3.27×10^{-6} from the limit 2.441406.

Observation 3.2

the product of sensitivity and temperature moves from its low-temperature value to half. Where x is large, $g(x)$ approaches x and $T \times S$ becomes 6539.895 K, set only by the difference in $1/\lambda$ between the two wavelengths, where x is small (the side of the Rayleigh-Jeans approximation), $g(x)$ approaches $1 + x/2$ and $T \times S$ becomes exactly half, 3269.947 K. The 4057.526 K at 40000 K lies in between.

4. Counting Differently

Computation 4.1: the per-frequency ratio at 3000, 5772, 10000 and 40000 K was 0.220760, 0.624407, 0.978518 and 1.425617, and at all seven temperatures it matched the per-wavelength ratio divided by $(550/440)^2 = 1.562500$ within 10^{-12} . The per-frequency sensitivity, by difference, was also the same as per wavelength within 10^{-6} .

Computation 4.2: temperature at which the two wavelengths are equally bright.

counting	temperature of equal brightness	peak at that temperature (as wavelength)
per wavelength	5904.132 K	490.804 nm
per frequency	10393.120 K	490.656 nm

The ratio of the two temperatures was 0.568081. For the peak positions, the root 4.965114232 of $x = 5(1 - e^{-x})$ ($hc/(kx) = 2.897771955 \times 10^{-3}$ m K) and the root 2.821439372 of $x = 3(1 - e^{-x})$ were used.

Observation 4.3

the value of the ratio and the temperature of equal brightness change with the way of counting, and the sensitivity does not. Per wavelength and per frequency, the ratios at the same two wavelengths differ by the constant 1.562500: a constant factor vanishes when the log is differentiated with respect to temperature, so the sensitivity is the same. On the other hand, “a star whose two colors are equally bright” is 5904.132 K per wavelength and 10393.120 K per frequency, and is not fixed unless one says which way was counted.

Computation 4.4: a pair set further apart (365 nm and 550 nm). The sensitivity at 6000, 20000 and 40000 K was 2.162614, 0.497335 and 0.209995, which is 2.045872, 2.087314 and 2.070174 times that of the 440 nm pair. The ratio of the differences in $1/\lambda$ between the two pairs was 2.027397, and the ratio of $T \times S$ at 10^8 K was 2.027420.

5. The Separator

Observation 5.1

the separator is whether the energy of one photon is above or below the thermal scale. Where x is large (10.899825 at 440 nm and 3000 K), the ratio responds to temperature exponentially through the difference in e^{-x} : once x becomes small (0.817487 at 440 nm and 40000 K), the ratio settles at the fourth power of the wavelength ratio, 2.441406. The sensitivity itself falls with temperature on both sides (Computation 2.1); what changes on entering this side is that the product of sensitivity and temperature halves from 6539.895 K to 3269.947 K (Computation 3.1).

Observation 5.2

what is not said. Actual filters and color index values, extinction by the atmosphere and interstellar dust, departures from a black body, the diagram of brightness against temperature, and the temperatures of individual stars are not treated. What is shown is that for two wavelengths of an ideal black body, the ratio approaches a fixed value and its sensitivity falls for hotter stars, and what changes and what does not with the way of counting was separated.

6. Roll Call and Closing

1. ratio R and sensitivity S (Section 1): a difference of $g(x)$.
2. **0.912395** of the limit at 40000 K (Computation 2.1): the ratio approaches a fixed value.
3. sensitivity **0.101438** and temperature error **9.858223** percent at 40000 K (Computation 2.1): hotter stars are harder to tell apart.
4. $T \times S$ from **6539.895** to **3269.947** (Computation 3.1): the coefficient moves to half.
5. equal brightness at **5904.132** K and **10393.120** K (Computation 4.2): changes with counting; the sensitivity does not.
6. above or below the thermal scale (Observation 5.1): the separator; the scope note of Paper 892; Papers 222 and 788.

Closing: The brightness ratio of two colors levels off for hotter stars, and its power to read temperature fades. The ratio of black-body brightness at 440 nm to 550 nm was already 0.912395 of the high-temperature limit 2.441406 at 40000 K, with a sensitivity of 0.101438, so a 1 percent error in the ratio became 9.858223 percent in temperature: the product of sensitivity and temperature moves from 6539.895 K at low temperature to exactly half, 3269.947 K. The value of the ratio and the temperature of equal brightness changed with the way of counting (5904.132 K and 10393.120 K), and the sensitivity did not. The separator is whether the energy of one photon is above or below the thermal scale. Placed among the earlier papers: the correspondence between a star's color and surface temperature that Paper 892 set aside was counted alongside the difference in counting from Paper 222 and the variable-independent dividing line from Paper 788. To be honest: how the temperatures of real stars are determined is not claimed. How well the ratio at two wavelengths of an ideal black body tells temperatures apart, and no further.

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Yuuki YAMAGISHI (山岸 勇輝)

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