

Soma Cube Solutions Lie in Only One of the Two Arrangements the Colour Count Allows

— only the V, T and P pieces carry a difference in chessboard colour, and two combinations add up to one, yet every solution falls in the same combination —the separator is whether one counts colour differences or actually searches by fitting the pieces — [Paper 944]

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Abstract

The Soma cube is a puzzle that fills a $3 \times 3 \times 3$ cube with one piece of three cubes and six pieces of four cubes. This paper counts all solutions, counts how each piece covers the two colours when the cube is coloured like a chessboard, and compares the combinations the colour count allows with those that actually have solutions. No new theorem or law is claimed. First, there are 11520 solutions counting orientation and position, 240 up to rotation and reflection, and none is its own image: up to rotation alone there are 480, and each of the 240 classes up to rotation and reflection consists of 48 solutions (Section 2). Second, only the V, T and P pieces carry a colour difference: colouring 14 even cells and 13 odd cells, L, Z, A and B always cover 2 of each colour, V has difference 1, and T and P have difference 2 (Section 3). Third, the colour count allows two combinations: the total difference +1 can only be made with V at +1 and T and P splitting +2 and −2 in opposite directions, two ways in all (Section 3). Fourth, all solutions lie in one combination and none in the other: all 11520 solutions have T at +2 and P at −2; searching with T at −2 and P at +2 examined 140415 nodes and found 0 solutions. Restricted to the combination with solutions, the search used 0.395194 of the nodes of the unrestricted search (Section 3). Fifth, the separator is whether one counts colour differences or actually searches by fitting the pieces: the colour count narrows the possible combinations to two but does not say which has solutions. That was settled by actually fitting the pieces in a search (Section 4). Keywords: Soma cube, polycube, chessboard colouring, packing puzzle, symmetry, exhaustive search.

Scope of this paper (scope note): No new theorem or law is claimed: the seven Soma pieces, the fact that there are 240 solutions (11520 counting orientation and position), and the idea of narrowing placements by chessboard colouring are all known. No reason is claimed: that the combination with T at -2 and P at $+2$ has no solution was only counted by search, and no explanation is given. Other shapes are not treated: building shapes other than $3 \times 3 \times 3$ and other sets of pieces are not treated. No method of solving is recommended: how people solve it and how hard it is are not discussed. Relation to earlier papers: Paper 925 showed that in peg solitaire an invariant sum narrows where the last peg can remain to five holes and that the peg can actually be left in all five. Here, of the two combinations the colour count allows, only one actually has solutions. Paper 847 showed that the count of shapes changes with whether reflections are identified. Here the number of solutions changes from 11520 with everything distinguished to 480 up to rotation and 240 up to reflection as well. What is added: counting all solutions and finding the classes under symmetry and their sizes, counting the colour difference of each piece, listing the two combinations the colour count allows and separating, by search, the one with solutions from the one without, comparing the number of search nodes, and placing the separator on “counting colour differences, or actually fitting the pieces”.

1. Notation and the Question

Pieces: V is three cubes in an L shape, L, T and Z are four cubes in a plane, and A, B and P are V with one cube placed on top. A and B coincide in a mirror and not under rotation. Shapes of four cubes number 8 when distinguished up to rotation and 7 when reflections are identified; L, T, Z, A, B and P are the 8 minus the straight line and the square (confirmed in the check).

Colour: colouring the $3 \times 3 \times 3$ cells by the parity of the coordinate sum gives 14 even cells (8 corners and 6 face centres) and 13 odd cells (12 edge midpoints and the centre). The colour difference of a piece is “even cells covered minus odd cells covered”. Summed over all pieces it is $14 - 13 = +1$.

Search: all placements of each piece are generated, and placements covering the lowest-numbered empty cell are tried in turn. All solutions are counted.

Question: how far does the colour count determine which combination the solutions lie in?

2. Number of Solutions and Symmetry

Computation 2.1: placements number V 144, L 144, T 72, Z 72, A 96, B 96 and P 64. Counting orientation and position there were 11520 solutions.

identification	solutions
all distinguished	11520
up to rotation (24)	480
up to rotation and reflection (48)	240

Observation 2.2

no solution is its own image. Each of the 240 classes up to rotation and reflection consists of 48 solutions, and $11520 \div 48 = 240.000000$: a mirror swaps A and B, so a reflected solution is still a solution with the same seven pieces. Every solution is carried to 47 others.

3. Colour Differences

Computation 3.1: how each piece covers the colours (even, odd).

piece	coverage	colour difference
L, Z, A, B	only 2, 2	0
V	2, 1 or 1, 2	+1 or -1
T	3, 1 or 1, 3	+2 or -2
P	3, 1 or 1, 3	+2 or -2

Exactly half of the placements of V, T and P have a positive difference (V 72 and 72, T 36 and 36, P 32 and 32).

Observation 3.2

the colour count allows two combinations. To make the differences of V, T and P add to +1, V must be +1, and T and P split +2 and -2 in opposite directions: the allowed combinations are $(V, T, P) = (+1, +2, -2)$ and $(+1, -2, +2)$.

Computation 3.3: combinations in the solutions, and searches restricted by combination.

search	solutions	nodes examined	nodes relative to unrestricted
unrestricted	11520	861285	1
V +1, T +2, P -2	11520	340375	0.395194
V +1, T -2, P +2	0	140415	0.163030

Observation 3.4

all solutions lie in one combination and none in the other. Looking at the 11520 solutions one by one, every combination was (+1, +2, -2): the colour count alone also allows the other combination, but a search restricted to it examined 140415 nodes and found 0 solutions.

4. The Separator

Observation 4.1

the separator is whether one counts colour differences or actually searches by fitting the pieces. Counting colour differences fixes the direction of V and narrows T and P to two ways, which of them has solutions does not follow from the colour count; only fitting the pieces in a search showed 11520 in one and 0 in the other.

Observation 4.2

what is not said. The reason the combination with T at -2 has no solution, shapes other than $3 \times 3 \times 3$, and how people solve the puzzle are not treated. What is shown is that of the two combinations the colour count allows, only one actually has solutions.

5. Roll Call and Closing

1. seven pieces and chessboard colour difference (Section 1): the total is +1.
2. 11520, 480 and 240 solutions (Computation 2.1): no solution is its own image.
3. only V, T and P carry a difference (Computation 3.1): ± 1 and ± 2 .

4. **two combinations allowed** (Observation 3.2): **T and P in opposite directions**.
5. **11520 in one, 0 in the other** (Computation 3.3): **node ratio 0.395194**.
6. **colour difference, or actual fitting** (Observation 4.1): **the separator; Paper 925; Paper 847**.

Closing: Soma cube solutions lie in only one of the two arrangements the colour count allows. There were 11520 solutions counting orientation and position and 240 up to rotation and reflection, and none was its own image: only V, T and P carry a chessboard colour difference, and two combinations make +1 with T and P in opposite directions, yet every solution had T at +2 and P at -2, and a search restricted to the other combination found 0 solutions. The separator is whether one counts colour differences or actually searches by fitting the pieces. Placed among the earlier papers: in Paper 925 every place an invariant sum allowed was actually reached, whereas here half of the combinations the colour count allows had no solution. The “count changes with identification” of Paper 847 appeared as 11520, 480 and 240. To be honest: the reason for the empty combination is not given. How far the colour count and the actual search agree, and where they part, and no further.

References

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