

# The Share of Legal Go Positions Shrinks by a Fixed Ratio per Point, and Less So on Wider Boards

— the rule that every group of stones must touch an empty point bites hard per point on a one-row board and more gently as the board widens —the separator is how many neighbours a point has — [Paper 945]

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## Abstract

When black stones, white stones and empty points are placed on a Go board, the arrangement is a position that can exist on the board (a legal position) if every group of stones is adjacent to an empty point. This paper counts legal arrangements on small boards in two ways, exhaustively and row by row, and finds how their share of all arrangements changes with board size and width. No new theorem or law is claimed. First, the two counts agreed on five boards: 57 on  $2 \times 2$ , 12675 on  $3 \times 3$ , 321689 on  $3 \times 4$ , 299681 on  $2 \times 6$  and 141209 on  $1 \times 12$  (Section 2). Second, the legal share falls as points are added, and differs with board shape for the same number of points: the share of all arrangements was 0.703704 on  $2 \times 2$ , 0.643957 on  $3 \times 3$  and 0.605315 on  $3 \times 4$ ; with the same 12 points,  $2 \times 6$  gave 0.563903 and  $1 \times 12$  gave **0.265710** (Section 2). Third, the growth from adding a row settles to a fixed ratio: for widths 1, 2, 3 and 4, the legal count grew by factors of 2.769292354, 8.533652512, 25.445014706 and 75.709341135 per added row, agreeing with the previous row to 9 digits (Section 3). Fourth, the per-point ratio is closer to 1 on wider boards: the per-point share (the factor by which the legal share changes when one point is added) was **0.923097** for width 1, 0.973747 for width 2, 0.980422 for width 3 and **0.983255** for width 4 (Section 3). Fifth, the separator is how many neighbours a point has: on a one-row board a point has at most two neighbours and groups rarely get an empty neighbour. As the board widens, neighbours increase and the same rule removes a smaller share per point (Section 4). **Keywords:** Go, legal positions, liberties, transfer matrix, enumeration, board width.

Scope of this paper (scope note): No new theorem or law is claimed: the definition of a legal Go position, counting row by row with a state, and the exponential growth of the number of legal positions with the number of points are all known. Only small boards are counted: only boards up to width 4 and exhaustive counts up to 12 points; the number for  $19 \times 19$  and limiting values for wide boards are not given. The rules of play are not entered: suicide, ko, the ban on repeated positions, and which positions appear in real games are not treated. Only arrangements are counted. Strength is not discussed: winning and losing and the quality of moves are not treated. Relation to earlier papers: Paper 846 showed that whether you can win is settled by painting backwards from the end, and how many moves it takes is not in that label. Here not winning or losing but the arrangements that can exist on the board are counted. What is added: checking the two counts against each other on five boards, listing the legal share by board size and shape, confirming that the ratio per added row settles, finding the per-point ratio for each width, and placing the separator on “how many neighbours a point has”.

## 1. Notation and the Question

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**Legal:** each point of the board holds black, white or empty. Stones of the same colour adjacent vertically or horizontally join into groups. An arrangement is called legal if every group is adjacent to at least one empty point.

**Counting:** (1) Exhaustive: all arrangements,  $3$  to the power of the number of points, are checked (up to 12 points). (2) Row by row: rows are added one at a time with a state holding the colours of the last row and, for its stones, which group each belongs to and whether that group already has an empty neighbour (a transfer matrix). An arrangement is discarded if a group does not continue into the next row and has no empty neighbour. The number of states was 5, 17, 67 and 277 for widths 1, 2, 3 and 4.

**Question:** what share of all arrangements is legal, and how does that share change with board size and width?

## 2. Counts on Small Boards

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**Computation 2.1:** exhaustive and row-by-row counts.

| board         | exhaustive | row by row | share of all arrangements |
|---------------|------------|------------|---------------------------|
| $2 \times 2$  | 57         | 57         | 0.703704                  |
| $3 \times 3$  | 12675      | 12675      | 0.643957                  |
| $3 \times 4$  | 321689     | 321689     | 0.605315                  |
| $2 \times 6$  | 299681     | 299681     | 0.563903                  |
| $1 \times 12$ | 141209     | 141209     | <b>0.265710</b>           |

The row-by-row count also agreed for boards turned sideways ( $2 \times 3$  and  $3 \times 2$ ,  $3 \times 4$  and  $4 \times 3$ ).

**Observation 2.2**

the more points, the lower the legal share, and for the same number of points the narrower board is lower. Of the three 12-point boards,  $3 \times 4$  gave 0.605315,  $2 \times 6$  gave 0.563903 and  $1 \times 12$  gave 0.265710: on a one-row board far more arrangements leave a group without an empty neighbour.

### 3. Ratio per Added Row and per Point

Computation 3.1: for width  $w$ , the ratio when the height grows by one (the last two of heights counted up to 40, 30, 24 and 18) and its  $w$ -th root (per point).

| width | ratio per added row | ratio per point | share relative to 3 |
|-------|---------------------|-----------------|---------------------|
| 1     | 2.769292354         | 2.769292        | <b>0.923097</b>     |
| 2     | 8.533652512         | 2.921242        | 0.973747            |
| 3     | 25.445014706        | 2.941266        | 0.980422            |
| 4     | 75.709341135        | 2.949765        | <b>0.983255</b>     |

For all four widths, the ratio per added row agreed with that of the previous row within a relative  $10^{-8}$  (it has settled as the height grows).

### Observation 3.2

the legal share falls by a nearly fixed ratio per point, and that ratio is closer to 1 on wider boards. The number of arrangements triples per point, but the legal count grows only by 2.769292 per point for width 1: the legal share becomes 0.923097 times as large per added point for width 1 and 0.983255 times for width 4. In the order of widths 1, 2, 3 and 4 the ratio approached 1.

## 4. The Separator

### Observation 4.1

the separator is how many neighbours a point has. The legality rule is only “every group has an empty neighbour”, the same on every board: on a one-row board a point has at most two neighbours, groups rarely get an empty neighbour, and a larger share is removed per added point. As the board widens, inner points have four neighbours and the same rule removes a smaller share per point.

### Observation 4.2

what is not said. The number for  $19 \times 19$ , the limiting ratio for wide boards, rules of play such as suicide and ko, and playing strength are not treated. What is shown is that on boards up to width 4 the legal share falls by a fixed ratio per point and that ratio approaches 1 as the width grows.

## 5. Roll Call and Closing

1. legality rule and two ways of counting (Section 1): **5, 17, 67 and 277 states.**
2. agreement on five boards (Computation 2.1): **12675 on  $3 \times 3$ .**
3. **0.265710** for  $1 \times 12$  with the same **12 points** (Computation 2.1): **narrower is lower.**
4. ratio per added row settles (Computation 3.1): **within a relative  $10^{-8}$ .**
5. per point from **0.923097** to **0.983255** (Computation 3.1): **closer to 1 on wider boards.**
6. number of neighbours of a point (Observation 4.1): **the separator; Paper 846.**

Closing: The share of legal Go positions shrinks by a fixed ratio per point, and less so on wider boards. The exhaustive and row-by-row counts agreed on five boards, and  $3 \times 3$  had 12675 legal arrangements, a share of

0.643957: per added point the legal share became 0.923097 times as large for width 1 and 0.983255 times for width 4, and with the same 12 points  $1 \times 12$  gave 0.265710 and  $3 \times 4$  gave 0.605315. The separator is how many neighbours a point has. Placed among the earlier papers: before the winning and losing that Paper 846 settled by painting backwards, the arrangements that can exist on the board were counted. To be honest: values for wide boards are not given. How much the legality rule removes from the arrangements on small boards, and how that changes with width, and no further.

## References

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1. Tromp, J., & Farnebäck, G. (2007). Combinatorics of Go. *Computers and Games*. Springer.
  2. Yamagishi, Y. (2026). *Whether You Can Win Is Settled by Painting Backwards; How Many Moves It Takes Is Not in That Label* [Paper 846]. YOI: <https://www.yamagishiyuuki.com/yoi/846>
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