

A Spinning Rope Is Not a Catenary: Shallow It Is Close to a Sine Arch, Wide Open Close to a Parabola

— the outward pull on the rope is proportional to its distance from the axis, its shape always lies farthest from the catenary that sags under the same force everywhere, and the tension rises the more it bows —the separator is whether the force bending the rope is proportional to distance from the axis or the same everywhere — [Paper 947]

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— *From thinking to syncing.* —

Abstract

When a rope held at both ends on the axis of rotation is spun fast, like a skipping rope, it bows out into an arc. This paper numerically solves the balance equation of a spinning rope without weight, compares its shape with a sine arch, a parabola and a catenary of the same length, and finds the tension required. No new theorem or law is claimed. First, in the shallow limit the rope is a sine arch and the tension approaches a fixed value: as the midpoint height is made small, the half-width of the shape was 1.570796326, matching $\pi/2 = 1.570796327$, and the tension became 0.405285 ($4/\pi^2$) times $\mu\omega^2 a^2$ (Section 2). Second, spun shallow, the sine arch is closest: with the length 1.05 times the spacing of the ends, the largest shape difference was 0.003160 (sine), 0.036839 (parabola) and 0.040748 (catenary) times the height (Section 3). Third, opened wide, the parabola becomes closer: with the length twice the spacing, the differences were 0.059168 (sine), 0.014000 (parabola) and 0.069937 (catenary). At all four lengths examined, the difference from the catenary was the largest (Section 3). Fourth, the tension rises the more the rope bows: with the length 1.05, 1.2, 1.5 and 2 times the spacing, the tension was 0.415386, 0.445331, 0.503784 and 0.597705 times $\mu\omega^2 a^2$. Integrating in angle and in horizontal position gave the half-width and length agreeing to 6 digits (Section 3). Fifth, the separator is whether the force bending the rope is proportional to distance from the axis or the same everywhere: a spinning rope is pulled outward more strongly at its middle, far from the axis, while a catenary is pulled by the same weight everywhere. How the force is distributed gives the same length of rope different shapes (Section 4).

Keywords: spinning rope, catenary, sine arch, centrifugal force, tension, troposkein.

Scope of this paper (scope note): No new theorem or law is claimed: the balance equation of a spinning rope, that its shape is a sine arch in the shallow limit, the shape of spinning ropes and vertical-axis wind turbine blades (called the troposkein), and the equations of the catenary and the parabola are all known. Weight is not included: weight, air resistance, and the rope's thickness and bending stiffness are not included. Only the limit of fast spinning, where centrifugal force is far larger than weight, is considered. No jumper is included: hand motion, the rope striking the ground and changes in spinning speed are not treated. No real rope is claimed: the ratios of length to spacing are set values and correspond to no skipping record. Relation to earlier papers: Paper 881, in its scope note, did not include time on the ground and did not include the time from landing to take-off, the slack of the rope, or air resistance; this paper sits in the seat of the slack of the rope (no earlier occupant). Paper 327 showed that the catenary and the parabola agree to within 0.083 per cent and are still not the same curve. Here a spinning rope is added, and it is counted that for the same length the catenary is always farthest and that whether the sine or the parabola is closer switches between shallow and deep. What is added: obtaining the shape by integrating the balance equation in two ways, checking the half-width and tension of the shallow limit against $\pi/2$ and $4/\pi^2$, listing the differences from three curves at four lengths, showing that the tension rises with bowing, and placing the separator on "force proportional to distance from the axis, or the same everywhere".

1. Notation and the Question

Setting: a rope of length L with its ends fixed on the axis of rotation at spacing $2a$ is spun at angular velocity ω . The linear density is μ and weight is ignored. In the rotating frame each part of the rope is pulled outward by a force $\mu\omega^2 y$ proportional to its distance y from the axis. The axial component H of the tension is the same along the rope, and the shape $y(x)$ follows the equation below.

$$H y'' = -\mu\omega^2 y \sqrt{1 + y'^2}$$

The curves for comparison are the catenary, sagging under the same force (weight) everywhere, $H y'' = \mu g \sqrt{1 + y'^2}$, and the parabola, with the same force per horizontal length, $H y'' = \text{constant}$. In the shallow limit $y'' \approx -(\mu\omega^2/H)y$ and the shape is a sine arch.

Method: starting from the middle (height h , slope 0), the equation is integrated until the rope returns to the axis in two ways: (1) integrating the tangent angle along the rope, and (2) integrating in horizontal position x . The h that gives the chosen ratio of length to spacing is found by bisection. The three comparison curves are given heights that make them the same length, and the largest height difference from the rope is found at 401 points.

Question: what shape does a spinning rope take, and how much tension does it need?

2. The Shallow Limit

Computation 2.1: integrating with midpoint height 10^{-4} (in units with $\mu\omega^2/H = 1$), the half-width of the shape was 1.570796326, matching $\pi/2 = 1.570796327$. The tension coefficient $H/(\mu\omega^2a^2)$ was 0.405285, matching $4/\pi^2 = 0.405285$.

Observation 2.2

in the shallow limit the shape is a sine arch and the tension is fixed at $4/\pi^2$. Because the outward force is proportional to distance from the axis, the equation of a shallow rope has the same form as sinusoidal oscillation: for that arch to fit exactly into the spacing $2a$, the tension must be 0.405285 times $\mu\omega^2a^2$.

3. Shape and Tension at Four Lengths

Computation 3.1: with half the spacing a set to 1: height, tension, and the largest shape difference from each of three curves of the same length divided by the rope's height.

length/spacing	rope height	tension $H/(\mu\omega^2a^2)$	difference from sine	from parabola	from catenary
1.05	0.289119	0.415386	0.003160	0.036839	0.040748
1.2	0.603949	0.445331	0.012825	0.031547	0.047507
1.5	1.031409	0.503784	0.031618	0.019711	0.057982
2	1.622828	0.597705	0.059168	0.014000	0.069937

The heights of the same-length curves were, for the sine, 0.289953, 0.609553, 1.048082 and 1.655581; for the parabola, 0.279861, 0.592682, 1.027675 and 1.634601; and for the catenary, 0.278608, 0.584688, 1.005267 and 1.592777.

Computation 3.2: half-width and half-length from the two integrations (in units with $\mu\omega^2/H = 1$).

length/spacing	half-width (angle, horizontal)	half-length (angle, horizontal)
1.05	1.551580, 1.551580	1.629159, 1.629159
1.2	1.498507, 1.498507	1.798208, 1.798208
1.5	1.408892, 1.408892	2.113338, 2.113338
2	1.293470, 1.293470	2.586940, 2.586940

Observation 3.3

shallow, the sine arch is closest; wide open, the parabola; the catenary is always farthest. At lengths 1.05 and 1.2 times the spacing the sine was closest, and at 1.5 and 2 times the parabola was closest: at all four lengths the difference from the catenary was the largest, 0.069937 times the height at 2 times. The first run assumed “the sine is closest at all four” and was wrong.

Observation 3.4

the tension rises the more the rope bows. From 0.405285 in the shallow limit, it became 0.597705 at a length twice the spacing: the first run assumed “the tension falls as it bows” and was wrong.

4. The Separator

Observation 4.1

the separator is whether the force bending the rope is proportional to distance from the axis or the same everywhere. A catenary is pulled by the same weight per length, and a parabola by the same force per horizontal length: a spinning rope is pulled harder the farther it is from the axis and hardly at all near its ends. For a shallow rope that distribution makes a sine arch (the shallow-limit equation). Why the shape approaches a parabola when opened wide is not checked here.

Observation 4.2

what is not said. Weight, air resistance, the rope's bending stiffness, the jumper's hand motion and real skipping speeds are not treated. What is shown is how far the shape of a rope spinning without weight differs from each of three curves, and that the tension rises as it bows.

5. Roll Call and Closing

1. balance equation of a spinning rope (Section 1): force proportional to distance from the axis.
2. shallow-limit half-width $\pi/2$ and tension $4/\pi^2$ (Computation 2.1): a sine arch.
3. difference **0.003160** from the sine at 1.05 times (Computation 3.1): shallow means sine.
4. difference **0.014000** from the parabola at 2 times (Computation 3.1): wide open means parabola; catenary always farthest.
5. tension from 0.405285 to **0.597705** (Computation 3.1): rises with bowing.
6. proportional to distance, or the same everywhere (Observation 4.1): the separator; the scope note of Paper 881; Paper 327.

Closing: A spinning rope is not a catenary: shallow it is close to a sine arch, wide open close to a parabola. Integrating the equation of a rope spinning without weight in two ways, the shallow-limit half-width was $\pi/2$ and the tension 0.405285 times $\mu\omega^2 a^2$: with the length 1.05 times the spacing of the ends the difference from the sine was 0.003160 times the height, at 2 times the difference from the parabola was 0.014000 times, and at all four lengths the difference from the catenary was the largest. At 2 times the tension rose to 0.597705 times. The separator is whether the force bending the rope is proportional to distance from the axis or the same everywhere. Placed among the earlier papers: the slack of the rope that Paper 881 left out was counted as the shape of a spinning rope, and a spinning rope was placed between the catenary and the parabola of Paper 327. To be honest: a real skipping rope also sags under its weight. In the limit without weight, how the distribution of force changes shape and tension, and no further.

References

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