

A Profit-Maximising Input Level Exists Only When the Exponents Sum to Less Than 1, and Profit Is 1 Minus That Sum of Revenue

— the closer the sum is to 1 the more sensitive the optimal scale is to price, at exactly 1 the choice splits into producing without limit or not at all, and above 1 there is no maximum —the separator is whether scale has an upper limit — [Paper 948]

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— *From thinking to syncing.* —

Abstract

When two inputs are combined as a product of powers and the output price and input prices are fixed, one can ask what input level maximises profit. This paper finds the profit maximum by formula and by numerical search for four exponent pairs summing to 0.5, 0.8, 0.9 and 0.95, counts the shares of cost and profit in revenue and the sensitivity to price, and compares cases where the exponents sum to 1 and 1.1. No new theorem or law is claimed. First, when the exponents sum to less than 1, one profit-maximising input level exists: for all four pairs, the maximum from the formula and from the numerical search agreed within a relative 10^{-6} (Section 2). Second, then each input's cost is exactly its exponent's share of revenue, and profit is 1 minus the sum: with exponents 0.3 and 0.6, costs were 0.300000 and 0.600000 of revenue and profit **0.100000**. With sums 0.5, 0.8 and 0.95, profit was 0.500000, 0.200000 and 0.050000 (Section 2). Third, the closer the sum is to 1, the more sensitive the optimal scale is to price: raising price by 1 per cent raised output by 1.000000, 4.000000, 9.000000 and **19.000000** per cent for sums 0.5, 0.8, 0.9 and 0.95 (Section 3). Fourth, at a sum of exactly 1, the choice splits into producing without limit or not at all: with exponents 0.5 and 0.5, scaling by t gave profit of +0.5 per unit t at price 2.5 (500.000000 at $t = 1000$) and -0.5 per unit t at price 1.5, with no maximum in between in either case (Section 4). Fifth, the separator is whether scale has an upper limit: with the sum at 1.1, scaling by 1, 10, 100 and 1000 gave profits of 0.500000, 11.473135, 196.223298 and **2988.155787**, still growing. A profit maximum exists only when widening scale stops the growth of earnings, that is, when the sum is below 1 (Section 5). **Keywords:** profit maximisation, product-of-powers production, returns to scale, cost shares, price sensitivity, production function.

Scope of this paper (scope note): No new theorem or law is claimed: the product-of-powers production function, the conditions for maximising profit at fixed prices, that cost shares equal the exponents, and that there is no profit maximum when the exponents sum to 1 or more are all known in economics. No real economy is claimed: the exponent pairs and prices are set values for comparison and correspond to no particular firm or industry. Prices are held fixed: selling and input prices are assumed not to change with the quantity produced. Market equilibrium, monopoly and taxes are not treated. Time is not included: the time to add capacity, borrowing and risk are not treated. Relation to earlier papers: Paper 934, in its scope note, did not treat prices and costs and did not treat how much to invest or the quantity that maximises profit; this paper sits in that seat (no earlier occupant). What is added: checking the formula maximum against numerical search for four exponent pairs, finding the shares of cost and profit in revenue from the exponents, finding the sensitivity to price for each sum, showing by varying scale that sums of 1 and 1.1 have no maximum, and placing the separator on “whether scale has an upper limit”.

1. Notation and the Question

Setting: two inputs x and y produce $Q = x^\alpha y^\beta$, sold at price p . With input prices r and w , profit is:

$$\pi = pQ - rx - wy, \quad s = \alpha + \beta$$

For $s < 1$, output and inputs at the profit maximum are

$$Q^* = \left[\left(\frac{p\alpha}{r} \right)^\alpha \left(\frac{p\beta}{w} \right)^\beta \right]^{1/(1-s)}, \quad x^* = \frac{p\alpha Q^*}{r}, \quad y^* = \frac{p\beta Q^*}{w}$$

The check sets $p = r = w = 1$ and also finds the maximum by a search that raises profit on $\log x$ and $\log y$ while halving the step.

Question: when does a profit-maximising input level exist, how is revenue divided there, and how sensitive is the optimal scale to price?

2. Profit Maximum and the Division of Revenue

Computation 2.1: profit maximum at $p = r = w = 1$.

exponents α, β	sum s	output Q^*	profit (formula)	profit (search)	profit/revenue
0.3, 0.2	0.5	0.255085	0.127542	0.127542	0.500000
0.4, 0.4	0.8	0.025600	0.005120	0.005120	0.200000
0.3, 0.6	0.9	1.259712×10^{-3}	0.000126	0.000126	0.100000
0.45, 0.5	0.95	7.389459×10^{-7}	—	—	0.050000

For all four pairs the search profit agreed with the formula within a relative 10^{-6} (for the sum 0.95 the values are small, so only the ratio is shown). Cost/revenue was 0.300000, 0.400000, 0.300000 and 0.450000 for x and 0.200000, 0.400000, 0.600000 and 0.500000 for y , equal to the exponents.

Observation 2.2

at the maximum, revenue is paid to the inputs in the shares of the exponents, and the remaining 1 minus the sum is profit. With exponents 0.3 and 0.6, 0.300000 of revenue goes to x , 0.600000 to y , and profit is 0.100000: the closer the sum is to 1, the smaller the profit share even at the maximum; at a sum of 0.95 it was 0.050000.

3. Sensitivity to Price

Computation 3.1: change in output (per cent) when price rises from $p = 1$ to 1.01, and $s/(1 - s)$.

sum of exponents	change in output	$s/(1 - s)$
0.5	1.000000	1.000000
0.8	4.000000	4.000000
0.9	9.000000	9.000000
0.95	19.000000	19.000000

Observation 3.2

the closer the sum is to 1, the more sensitive the optimal scale is to price. A 1 per cent rise in price raises output by 1 per cent at a sum of 0.5 but by 19 per cent at 0.95: the change in output matched $s/(1-s)$, which grows without bound as the sum approaches 1.

4. Sums of Exactly 1 and Above 1

Computation 4.1: profit with $x = y = t$ at scale $t = 1, 10, 100$ and 1000.

exponents, price	$t = 1$	$t = 10$	$t = 100$	$t = 1000$
0.5, 0.5, price 2.5	0.500000	5.000000	50.000000	500.000000
0.5, 0.5, price 1.5	−0.500000	−5.000000	−50.000000	−500.000000
0.5, 0.6, price 2.5	0.500000	11.473135	196.223298	2988.155787

Observation 4.2

at a sum of 1, profit is proportional to scale and there is no maximum in between. At price 2.5 profit is +0.5 per unit t and grows as scale widens; at price 1.5 it is −0.5 per unit t and producing nothing is best: at a sum of 1.1, profit per unit t also kept growing with scale.

5. The Separator

Observation 5.1

the separator is whether scale has an upper limit. With the sum below 1, the growth of earnings stops as scale widens and a single profit maximum exists: with the sum at 1 the growth is constant, and depending on price the answer is “without limit” or “not at all”; above 1 the growth accelerates and there is no maximum. The difference in the sum of exponents that Paper 934 counted appeared here as the presence or absence of a profit maximum.

Observation 5.2

what is not said. Real firms, prices that change with quantity, market equilibrium, time and risk are not treated. What is shown is that with product-of-powers production at fixed prices, whether a profit maximum exists, how revenue is divided and the sensitivity to price are set by the sum of exponents.

6. Roll Call and Closing

1. product-of-powers production and profit (Section 1): sum of exponents s .
2. formula and search maxima agree (Computation 2.1): a maximum exists for $s < 1$.
3. profit/revenue **0.100000** (sum 0.9) (Computation 2.1): costs in exponent shares.
4. output up **19.000000** per cent for a 1 per cent price rise (sum 0.95) (Computation 3.1): $s/(1-s)$.
5. ± 0.5 per unit t at sum 1, **2988.155787** at 1.1 (Computation 4.1): no maximum.
6. whether scale has an upper limit (Observation 5.1): the separator; the scope note of Paper 934.

Closing: A profit-maximising input level exists only when the exponents sum to less than 1, and profit is 1 minus that sum of revenue. For four pairs summing to 0.5, 0.8, 0.9 and 0.95, the formula and search maxima agreed, costs took the exponents' shares of revenue, and profit was 0.500000, 0.200000, 0.100000 and 0.050000: raising price by 1 per cent raised output by 1, 4, 9 and 19 per cent; at a sum of 1 profit was proportional to scale with no maximum, and at 1.1 it kept growing. The separator is whether scale has an upper limit. Placed among the earlier papers: the sum of exponents that Paper 934 counted on the production side alone was counted, with prices and costs added, as the presence or absence of a profit maximum. To be honest: the practice of deciding how much to produce is not claimed. In a model with fixed prices, how the sum of exponents sets where the profit maximum lies and its sensitivity, and no further.

References

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