

Spherical Aberration Gives the Eye a Middle-Sized Best Pupil That Barely Moves with the Aberration's Strength

— widening the pupil reduces diffraction blur, but aberration at the rim grows as the fourth power of the diameter, so contrast at the Nyquist fineness peaks at a middle-sized pupil —the separator is whether the gain from widening the pupil or the loss from rim aberration wins — [Paper 949]

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— *From thinking to syncing.* —

Abstract

Considering diffraction alone, the larger the pupil, the more contrast it passes for fine gratings. This paper adds to the eye values of Paper 919 a spherical aberration that grows toward the rim of the pupil, and numerically computes, for each pupil diameter, the contrast of the finest grating the cone mosaic can pick up (the Nyquist fineness) with defocus chosen for best contrast. No new theorem or law is claimed. First, without aberration, the larger the pupil the better: the numerical computation reproduced the diffraction-only formula (0.266568 against 0.266680 at a 3 mm pupil, 0.612981 against 0.612983 at 6 mm), and there was no interior maximum up to the 8 mm examined (0.707238 at 8 mm) (Section 2). Second, with aberration, the best pupil settles at a middle size: setting the rim aberration at a 6 mm pupil to 0.5, 1, 2 and 4 wavelengths, the best pupils were 6.149, 5.207, 4.523 and **3.970** mm, with contrasts 0.573231, 0.511423, 0.445752 and 0.378165 (Section 3). Third, the best pupil barely responds to the strength of the aberration: making the aberration 8 times stronger shrank the best pupil only to **0.645625** times its size. The power relating diameter to aberration strength was -0.210410 (Section 3). Fourth, for small pupils aberration hardly matters: at a 2.5 mm pupil the contrast changed only from 0.151378 to 0.151299 as aberration went from 0 to 4 wavelengths (Section 3). Fifth, the separator is whether the gain from widening the pupil or the loss from rim aberration wins: widening the pupil reduces diffraction blur, but rim aberration grows as the fourth power of the diameter. The gain wins for small pupils, the loss for large ones, and the boundary between them is the best pupil (Section 4). **Keywords:** spherical aberration, pupil diameter, modulation transfer function, diffraction, Nyquist fineness, optics of the eye.

Scope of this paper (scope note): No new theorem or law is claimed: obtaining the optical transfer function as the autocorrelation of the pupil function, the modulation transfer function formula for a diffraction-only circular pupil, that spherical aberration grows as the fourth power of position measured to the rim, and that balancing aberration with defocus improves imaging are all known. These are set values: wavelength, cone spacing and nodal distance are as in Paper 919, and the strength of spherical aberration is a set value for comparison. It corresponds to no one's eye. Other aberrations and the retina are not included: chromatic aberration, astigmatism, the tear film, scatter, and the retina's different response to light entering through different parts of the pupil are not included. No visual acuity is predicted: contrast is a value for showing the comparison; anyone's acuity and changes of pupil with brightness are not treated. Relation to earlier papers: Paper 919, in its scope note, did not include aberrations, leaving out the eye's spherical aberration, chromatic aberration, tear film and scatter, and set up diffraction-only optics; this paper sits in the seat of spherical aberration (no earlier occupant). Paper 358, in its scope note, also did not build optics and did not enter the theory of diffraction limits, aberrations or point spread functions; this paper also sits in that seat of aberrations. Paper 919 showed that with the pupil wider than 1.85 mm, the eye's limit of fineness is set not by diffraction but by the spacing of cones. Here it is counted how aberration erodes the contrast at that limit as the pupil opens further. What is added: reproducing the diffraction-only formula numerically, finding the best pupil and its contrast for four aberration strengths, showing by a power that the best pupil barely responds to aberration strength, setting out that aberration hardly matters for small pupils, and placing the separator on "whether the gain from widening the pupil or the loss from rim aberration wins".

1. Notation and the Question

Setting: as in Paper 919, the wavelength is 555 nm, cone spacing 2.5 μm and the nodal point to retina distance 16.7 mm. The finest grating the cone mosaic can pick up (the Nyquist fineness) is 58.2940 cycles per degree. With pupil diameter D and position ρ normalised to 1 at the rim, the wavefront error is set as follows.

$$W(\rho) = W_{020} \rho^2 + W_{040} \rho^4, \quad W_{040} = A \left(\frac{D}{6 \text{ mm}} \right)^4 \text{ (wavelengths)}$$

A is the rim spherical aberration at a 6 mm pupil and W_{020} is defocus, chosen for each pupil to give the highest contrast. The modulation transfer function (contrast) at the Nyquist fineness is computed numerically as the magnitude of the autocorrelation of the laterally shifted pupil function.

Question: with spherical aberration, which pupil gives the highest contrast at the Nyquist fineness, and how does it move with the strength of the aberration?

2. Without Aberration

Computation 2.1: numerical values without aberration and the diffraction-only formula.

pupil	numerical	formula (values of Paper 919)
3 mm	0.266568	0.266680
6 mm	0.612981	0.612983

Contrasts for pupils of 1.5, 2, 2.5, 3, 3.5, 4, 5, 6, 7 and 8 mm were 0.0000, 0.0235, 0.1514, 0.2666, 0.3587, 0.4318, 0.5385, 0.6130, 0.6669 and 0.7072; the highest was at the edge of the range examined, 8 mm (0.707238).

Observation 2.2

without aberration, the larger the pupil the better, with no interior maximum. The numerical computation reproduced the diffraction-only formula within a difference of 5×10^{-3} at 3 mm and 6 mm. This is the starting point for the computation with aberration.

3. With Aberration

Computation 3.1: contrast by pupil (with defocus chosen for best contrast).

rim aberration <i>A</i>	3 mm	4 mm	5 mm	6 mm	7 mm	8 mm
0.5	0.2665	0.4309	0.5305	0.5729	0.5299	0.3660
1	0.2665	0.4283	0.5071	0.4646	0.2505	0.2116
2	0.2661	0.4178	0.4215	0.2257	0.1840	0.1372
4	0.2648	0.3781	0.1941	0.1628	0.1112	0.0859

Computation 3.2: best pupil (narrowed by golden-section search).

rim aberration A	best pupil	contrast	defocus W_{020} (wavelengths)
0.5	6.149 mm	0.573231	−0.5068
1	5.207 mm	0.511423	−0.5110
2	4.523 mm	0.445752	−0.5809
4	3.970 mm	0.378165	−0.7025

Increasing the aberration eightfold from 0.5 to 4 made the best pupil **0.645625** times as large, a power of −0.210410 (−0.239947 between 0.5 and 1, −0.203297 between 1 and 2). At a 2.5 mm pupil the contrast for aberrations 0, 0.5, 1, 2 and 4 was 0.151378, 0.151377, 0.151373, 0.151358 and 0.151299.

Observation 3.3

the best pupil settles at a middle size and barely responds to aberration strength. Even with eight times the rim aberration, the best pupil shrank only from 6.149 mm to 3.970 mm: for small pupils the aberration strength hardly matters, and at 2.5 mm the contrast was 0.151299 even at 4 wavelengths. The best defocus was opposite in sign to the aberration, from −0.5068 to −0.7025 wavelengths.

4. The Separator

Observation 4.1

the separator is whether the gain from widening the pupil or the loss from rim aberration wins. Widening the pupil reduces diffraction blur and raises contrast at the Nyquist fineness: meanwhile rim aberration grows as the fourth power of the diameter. Without aberration only the gain remains and the larger pupil is better; with aberration the loss wins for large pupils, and the boundary became a middle-sized best pupil.

Observation 4.2

what is not said. The aberration strength of real eyes, chromatic aberration, astigmatism, tear film and scatter, retinal response, changes of pupil with brightness, and anyone's acuity are not treated. What is shown is that under a set spherical aberration the pupil maximising contrast at the Nyquist fineness settles at a middle size, and that size barely responds to aberration strength.

5. Roll Call and Closing

1. wavefront error and numerical transfer function (Section 1): **rim aberration as the fourth power of diameter.**
2. formula reproduced without aberration (Computation 2.1): **the larger the pupil the better.**
3. best pupil from 6.149 to **3.970 mm** (Computation 3.2): **settles at a middle size.**
4. **0.645625** times for eightfold aberration (Computation 3.2): **power -0.210410 .**
5. 0.151378 to 0.151299 at 2.5 mm (Computation 3.2): **aberration does not matter for small pupils.**
6. gain from widening, or loss at the rim (Observation 4.1): **the separator; the scope notes of Papers 919 and 358.**

Closing: Spherical aberration gives the eye a middle-sized best pupil that barely moves with the aberration's strength. With diffraction alone the larger pupil was better, with contrast 0.707238 at 8 mm: setting the rim aberration to 0.5, 1, 2 and 4 wavelengths, the best pupils were 6.149, 5.207, 4.523 and 3.970 mm, and eight times the aberration made the pupil only 0.645625 times as large. At a 2.5 mm pupil aberration hardly mattered. The separator is whether the gain from widening the pupil or the loss from rim aberration wins. Placed among the earlier papers: to the limit of the eye's fineness that Paper 919 set with diffraction alone, the aberration that Papers 919 and 358 left out was added, and how the contrast at that limit is eroded as the pupil opens was counted. To be honest: a real eye also has chromatic aberration and scatter. Under a set spherical aberration, where the best pupil settles and how far it moves, and no further.

References

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