

In a Flow-Through Culture the Species That Breaks Even on Less Food Wins, and the Winner Switches with the Flow Rate

— even at flow rates where either species could persist alone, the remaining food falls to the winner's break-even concentration and the other loses to the flow —the separator is whether species are compared by maximum growth rate or by the food concentration at which they break even — [Paper 953]

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Abstract

In a flow-through culture (a chemostat), only species that can grow faster than they are washed out remain. This paper puts the species of Paper 933 and a species with a slower maximum growth rate that still grows on dilute food into the same vessel, and numerically finds which remains at each flow rate and what the food concentration becomes. No new theorem or law is claimed. First, the winner switches with the flow rate: with species A (maximum 0.40 /h, half-saturation 0.50 g/L) and species B (0.25 /h, 0.05 g/L), B remained at flow rates 0.10 and 0.20 /h and A at 0.24 and 0.30 /h. The break-even food concentrations are equal at **0.233333** /h (Section 2). Second, the remaining food always settles at the winner's break-even concentration: after 3000 hours the food was 0.033333, 0.200000, 0.750000 and 1.500000 g/L, matching the winner's break-even concentration within a relative 10^{-6} in all four cases (Section 2). Third, a species that could persist alone loses with a rival: at a flow rate of 0.24 /h, B alone could break even at 1.200000 g/L, but A brought the food down to 0.750000 g/L and B fell to 3.767166×10^{-9} (Section 2). Fourth, the slower species wins because it grows faster on dilute food: the two growth rates are equal at 0.700000 g/L of food (0.233333 /h), and below that B is faster. At 0.2 g/L, A grows at 0.114286 /h and B at **0.200000** /h (Section 3). Fifth, the separator is whether species are compared by maximum growth rate or by the food concentration at which they break even: the maximum growth rate is the speed with plenty of food, but in a flow-through culture the winning species makes the food dilute. What decides the winner is the food concentration needed to grow as fast as the flow (Section 4). **Keywords:** chemostat, resource competition, break-even concentration, competitive exclusion, dilution rate, Monod equation.

Scope of this paper (scope note): No new theorem or law is claimed: the Monod equation, the balance of a flow-through culture, and that among species competing for one food only the species with the lowest break-even concentration remains (resource competition theory) are all known. These are set values: species A and the incoming food concentration of 10.0 g/L are as in Paper 933, and species B is set for comparison. They correspond to no alga or bacterium. One food: more than one food, species producing toxins against each other, death and attachment to walls are not included. Coexistence is not discussed: only four flow rates are examined; what happens exactly where the break-even concentrations are equal is not treated. Relation to earlier papers: Paper 933, in its scope note, did not include death or attachment, leaving out the death of cells, attachment to walls and competition between several species; this paper sits in the seat of competition between several species (no earlier occupant). Paper 933 showed that in a flow-through culture the washout boundary is set not by the maximum growth rate but by the incoming feed, and that output peaks short of it. Here it is counted that with two species, what decides which remains is not the maximum growth rate either. Paper 680 showed that what is frightening about exponential growth is that half of it sits in the last doubling: the values of species A are its representative values. What is added: finding the break-even concentrations of two species and the flow rate at which they are equal, numerically finding the winner and remaining food at four flow rates, showing a case where a species that could persist alone loses, setting out the time for the loser to decline, and placing the separator on “maximum growth rate, or break-even food concentration”.

1. Notation and the Question

Setting: two species X_A and X_B are put into a vessel with incoming food concentration $S_0 = 10.0$ g/L and flow rate (dilution rate) D . Growth follows the Monod equation, with yield 0.5 for both.

$$\frac{dS}{dt} = D(S_0 - S) - \sum_i \frac{\mu_i(S)X_i}{Y}, \quad \frac{dX_i}{dt} = (\mu_i(S) - D)X_i, \quad \mu_i(S) = \frac{\mu_{\max,i}S}{K_i + S}$$

Species A has $\mu_{\max} = 0.40$ /h and $K = 0.50$ g/L (as in Paper 933), species B 0.25 /h and 0.05 g/L (set values). The food concentration a species needs to break even with the flow alone is $R_i^* = K_i D / (\mu_{\max,i} - D)$.

Method: starting with 0.05 g/L of each and food at S_0 , the system is followed to 3000 hours by Runge-Kutta with a step of 0.02 hours. The time at which the loser falls below 1/1000 of the winner is also recorded.

Question: when two species are put in the same vessel, which remains, and how does that change with the flow rate?

2. Four Flow Rates

Computation 2.1: break-even concentrations and the vessel after 3000 hours.

D (/h)	R_A^*	R_B^*	food (g/L)	A	B	time loser fell below 1/1000
0.10	0.166667	0.033333	0.033333	0.000000	4.983333	124.0 h
0.20	0.500000	0.200000	0.200000	0.000000	4.900000	220.5 h
0.24	0.750000	1.200000	0.750000	4.625000	3.767166×10^{-9}	507.6 h
0.30	1.500000	cannot grow	1.500000	4.250000	1.631820×10^{-77}	54.0 h

The break-even concentrations are equal at a flow rate of **0.233333** /h ($R^* = 0.700000$ g/L). In all four cases the surviving species had the smaller R^* , the food matched that R^* within a relative 10^{-6} , the surviving amount matched yield times ($S_0 - R^*$) within a relative 10^{-6} , and the loser was below 10^{-6} of the winner.

Observation 2.2

the species with the lower break-even concentration remains, and which one switches with the flow rate. Below 0.233333 /h B remained, above it A: at $D = 0.24$, B alone could break even at 1.200000 g/L, but A lowered the food to 0.750000 g/L and B's growth could no longer keep up with the flow. The time for the loser to die out was long near the boundary, 507.6 h at 0.24.

3. Why the Slower Species Can Win

Computation 3.1: the growth rates of the two species are equal at a food concentration of 0.700000 g/L, with growth rate 0.233333 /h. At 0.2 g/L of food, A grows at 0.114286 /h and B at **0.200000** /h. This concentration and growth rate coincide with the point where R^* are equal (0.7 g/L and 0.233333 /h).

Observation 3.2

B, with the slower maximum growth rate, grows faster than A on dilute food. Above 0.700000 g/L A is faster; below it B is faster: food in a flow-through culture is made dilute by the surviving species, so the slower the flow and the more dilute the break-even food, the more B, strong on dilute food, was favoured.

4. The Separator

Observation 4.1

the separator is whether species are compared by maximum growth rate or by the food concentration at which they break even. The maximum growth rate is the speed with plenty of food: in a flow-through culture the surviving species lowers the food to its own R^* , so the winner was decided by “the food concentration needed to grow as fast as the flow”. A, with the faster maximum, won only for $D > 0.233333$ /h, where A’s concentration was also the lower.

Observation 4.2

what is not said. Real species, more than one food, toxins, death and attachment, and behaviour exactly where R^* are equal are not treated. What is shown is that for two set species the surviving species and remaining food are decided by the break-even concentration and switch with the flow rate.

5. Roll Call and Closing

1. Monod equations for two species and R^* (Section 1): **flow rate and break-even food.**
2. R^* equal at **0.233333** /h (Computation 2.1): **B when slower, A when faster.**
3. remaining food is the winner’s R^* (Computation 2.1): **within a relative 10^{-6} .**
4. B down to 3.767166×10^{-9} at $D = 0.24$ (Computation 2.1): **a species that could persist alone loses.**
5. B at **0.200000** /h on 0.2 g/L of food (Computation 3.1): **the slower species is faster on dilute food.**
6. maximum growth rate, or break-even food (Observation 4.1): **the separator; the scope note of Paper 933; Paper 680.**

Closing: In a flow-through culture the species that breaks even on less food wins, and the winner switches with the flow rate. Putting A, with the faster maximum growth rate, and B, strong on dilute food, into the same vessel, B remained below a flow rate of 0.233333 /h and A above it: the remaining food matched the winner’s R^* in all four cases, and at $D = 0.24$ even B, which could persist alone, fell to 3.767166×10^{-9} . The separator is whether species are compared by maximum growth rate or by the food concentration at which they break even. Placed among the earlier papers. What Paper 933 showed for one species, that the boundary is not set by the maximum growth rate, was extended to which of two species wins. To be honest: the turnover of species in lakes and bays is not claimed. What decides the survivor among two species competing for one food, and no further.

References

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