

Guessing Shifts the Level an Up-Down Method Tracks Downward, and a One-Down Rule Has No Convergence Point in Two-Choice Tasks

— in choice tasks the proportion correct has a guessing floor, so for the same target proportion the stimulus level that reaches it falls by the amount of the floor —the separator is whether the floor of proportion correct lies above or below the rule’s target — [Paper 954]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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YOI: <https://www.yamagishiyuuki.com/yoi/954>

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Abstract

An up-down method weakens the stimulus after a correct answer and strengthens it after an error, tracking the point of a fixed proportion correct. This paper adds guessing to the observer of Paper 935 and finds, by formula and with random observers, where rules that step down after 1, 2 or 3 correct answers in a row converge in two-choice and four-choice tasks. No new theorem or law is claimed. First, with guessing, the level the same rule tracks shifts downward: the target of the two-down rule is 0.544952 in the yes-no task and **−0.216719** in two-choice, 0.761671 apart. The random observer converged to 0.558854 and **−0.224659** (Section 2). Second, the one-down rule has no target in two-choice tasks: its target proportion 0.5 equals the two-choice guessing floor, which is reached however weak the stimulus. The random observer fell to **−2.680489**, with a proportion correct of 0.513672 in the second half (Section 2). Third, a rule that does not converge spreads out as trials increase: for the one-down rule in two-choice, increasing trials from 600 to 2400 widened the spread of final positions by **2.033179** times, while the two-down rule widened only 1.039756 times (Section 3). Fourth, with the lower floor of four-choice, even the one-down rule has a target: in four-choice (guessing 0.25) the targets of the one-, two- and three-down rules are **−0.430727**, 0.277953 and 0.597562, and the random observer converged to **−0.454345**, 0.284358 and 0.611233. For all eight combinations with a target, the second-half proportion correct was within 0.002 of the target (Section 2). Fifth, the separator is whether the floor of proportion correct lies above or below the rule’s target: if the floor is below the target, the rule converges at a level lowered by the floor. If the floor is at or above the target, there is no convergence point, and the stimulus wanders on the weak side (Section 4). **Keywords:** up-down method, guessing, forced choice, psychometric function, proportion correct, random observer.

Scope of this paper (scope note): No new theorem or law is claimed: up-down methods and their transformed versions, that proportion correct in choice tasks starts from the guessing rate, and that a k -down rule tracks the k -th root of proportion correct 0.5 are all known. The model of Paper 935: the observer answers correctly at level x with a proportion mixing guessing γ and the standard normal cumulative distribution; fatigue, adaptation and lapses are not included. No judgement of rules: estimation efficiency, choice of trial numbers, and which rule to use are not treated. No limit is claimed: the spread of a non-converging rule is measured up to 2400 trials; behaviour as trials increase without bound is not claimed. Relation to earlier papers: Paper 935, in its scope note, did not treat actual perception and did not include choice tasks with guessing, fatigue or adaptation; this paper sits in the seat of choice tasks with guessing (no earlier occupant). Paper 935 showed that the proportion an up-down method tracks is chosen by its rule, and that with an asymmetric rule the converged level shifts up in proportion to the step. Here it is counted that with a guessing floor the same rule tracks a lower level, and that when the floor reaches the target the convergence point itself disappears. What is added: finding by formula the targets of three rules in yes-no, two-choice and four-choice tasks, confirming convergence points and proportions correct with random observers, showing that the spread of a non-converging rule widens with trials, and placing the separator on “whether the floor of proportion correct lies above or below the rule’s target”.

1. Notation and the Question

Observer: the proportion correct at stimulus level x is set as follows. γ is the guessing rate: 0 for the yes-no task (Paper 935), 0.5 for two-choice and 0.25 for four-choice.

$$p(x) = \gamma + (1 - \gamma) \Phi(x)$$

Rule: after k correct answers in a row, lower x by a step of 0.1; after any error, raise it. The rule converges where proportion correct is $0.5^{1/k}$, so the target level is the x solving $p(x) = 0.5^{1/k}$. If $0.5^{1/k} \leq \gamma$ there is no solution.

Random observer: as in Paper 935, start at 2.0, step 0.1, 1000 runs of 600 trials, averaging the level over the last 400 trials.

Question: in tasks with guessing, where does the same rule converge, and when does it fail to converge?

2. Convergence Points

Computation 2.1: target level, the random observer’s convergence point (standard error in parentheses) and second-half proportion correct.

task, rule	target proportion	target level	converged level	proportion correct
yes-no, 1-down	0.500000	−0.000000	−0.002298 (0.001934)	0.500108
yes-no, 2-down	0.707107	0.544952	0.558854 (0.002130)	0.706233
yes-no, 3-down	0.793701	0.819329	0.834829 (0.002226)	0.793065
two-choice, 1-down	0.500000	none	−2.680489 (0.028605)	0.513672
two-choice, 2-down	0.707107	−0.216719	−0.224659 (0.003744)	0.707117
two-choice, 3-down	0.793701	0.220865	0.234317 (0.003400)	0.793233
four-choice, 1-down	0.500000	−0.430727	−0.454345 (0.002886)	0.500062
four-choice, 2-down	0.707107	0.277953	0.284358 (0.002554)	0.706425
four-choice, 3-down	0.793701	0.597562	0.611233 (0.002473)	0.792843

For all eight combinations with a target, the converged level was within a quarter of the step of the target and the second-half proportion correct within 0.002. The targets of the two-down rule differ by 0.761671 between yes-no and two-choice.

Observation 2.2

the same rule tracks a level lowered by the guessing floor. The two-down rule tracks proportion correct 0.707107 in every task, but in two-choice there is a floor of 0.5, so the level reaching it falls to −0.216719: for the one-down rule in two-choice, the target 0.5 equals the floor, and the random observer fell to −2.680489 with proportion correct still 0.513672.

3. Spread of a Rule That Does Not Converge

Computation 3.1: two-choice, final positions over 400 runs each.

rule, trials	mean position	spread (standard deviation)	spread ratio for 4 times the trials
1-down, 600	−3.130000	1.438915	
1-down, 2400	−5.169500	2.925571	2.033179
2-down, 600	−0.213000	0.288077	
2-down, 2400	−0.234000	0.299530	1.039756

Observation 3.2

a rule that does not converge spreads out as trials increase. With four times the trials, the two-down rule hardly changed in mean or spread: the one-down rule's mean kept falling from −3.130000 to −5.169500 and its spread widened 2.033179 times. On the weak side the proportion correct is close to the floor of 0.5 (0.513672 in the second half of 600 trials), so a step down is hardly more likely than a step up.

4. The Separator

Observation 4.1

the separator is whether the floor of proportion correct lies above or below the rule's target. What the rule chooses is a proportion correct, not a stimulus level: if the floor is below the target, there is one level reaching that proportion, and the rule converges there, lowered by the floor. If the floor equals the target, weakening the stimulus never moves the proportion away from the target, and the convergence point disappeared. The four-choice floor of 0.25 is below the target 0.5, so even the one-down rule had a target at −0.430727.

Observation 4.2

what is not said. Real perception, fatigue, adaptation and lapses, estimation efficiency, which rule to use, and behaviour as trials increase without bound are not treated. What is shown is that a guessing floor lowers the level the same rule tracks, and that when the floor reaches the target the convergence point disappears.

5. Roll Call and Closing

1. proportion correct with guessing and rule targets (Section 1): solve for $0.5^{1/k}$.
2. **−0.216719** for two-choice, 2-down (Computation 2.1): **0.761671** below yes-no.
3. no target for two-choice, 1-down (Computation 2.1): falls to **−2.680489**.
4. spread **2.033179** times for 4 times the trials (Computation 3.1): does not converge.
5. four-choice 1-down has a target at **−0.430727** (Computation 2.1): lower floor.
6. floor above or below the target (Observation 4.1): the separator; the scope note of Paper 935.

Closing: Guessing shifts the level an up-down method tracks downward, and a one-down rule has no convergence point in two-choice tasks. The target of the two-down rule was 0.544952 in yes-no and **−0.216719** in two-choice, and random observers converged there: for the one-down rule in two-choice, the target coincided with the floor, and four times the trials widened the spread of positions 2.033179 times. In four-choice, with a lower floor, even the one-down rule had a target. The separator is whether the floor of proportion correct lies above or below the rule's target. Placed among the earlier papers: the guessing floor that Paper 935 left out was added to the up-down method in which Paper 935 showed that “the rule chooses the proportion”. To be honest, which rule to use is not claimed. Whether and where a rule converges, given the positions of floor and target, and no further.

References

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